

The effective action of Type IIA Calabi-Yau orientifolds¹

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ABSTRACT

The $N = 1$ effective action for generic type IIA Calabi-Yau orientifolds in the presence of background fluxes is computed from a Kaluza-Klein reduction. The Kähler potential, the gauge kinetic functions and the flux-induced superpotential are determined in terms of geometrical data of the Calabi-Yau orientifold and the background fluxes. The moduli space is found to be a Kähler subspace of the $N = 2$ moduli space and shown to coincide with the moduli space arising in compactification of M-theory on a specific class of G_2 manifolds. The superpotential depends on all geometrical moduli and vanishes at leading order when background fluxes are turned off. The $N = 1$ chiral coordinates linearize the appropriate instanton actions such that instanton effects can lead to holomorphic corrections of the superpotential. Mirror symmetry between type IIA and type IIB orientifolds is shown to hold at the level of the effective action in the large volume – large complex structure limit.

December 2004

¹Work supported by: DFG – The German Science Foundation, European RTN Program MRTN-CT-2004-503369 and the DAAD – the German Academic Exchange Service.

1 Introduction

Compactifications of type II string theories with space-time filling D-branes and background fluxes are currently under investigation. The reason is that they lead to phenomenologically interesting string vacua both for particle physics as well as for cosmology [1, 2]. If the string background includes a compact internal manifold Y , consistency requires that apart from D-branes also negative tension objects have to be present. Such objects are known as orientifold planes and they arise when the string theory is modded out by a discrete symmetry which includes parity reversal of the worldsheet [3, 4, 5].

From a phenomenological point of view spontaneously broken $N = 1$ vacua are of particular interest. They can arise by first compactifying type II string theories on specific orientifolds of Calabi-Yau manifolds which preserve one of the two supersymmetries present in standard Calabi-Yau compactifications of type II theories [6, 7, 8, 9, 10]. The D-branes can then be arranged such that they preserve the same supersymmetry. These string vacua realize an $N = 1$ supersymmetry which is spontaneously broken once background fluxes are turned on [11, 12, 13].

In order to discuss the phenomenological properties of such vacua in some detail it is essential to determine the low energy effective action and in particular the couplings of the bulk moduli [14]. In this paper we focus on type IIA string theory compactified on generic Calabi-Yau orientifolds and determine its low energy effective action in terms of geometrical data of the Calabi-Yau orientifold and the background fluxes. Specifically we determine the Kähler potential, the superpotential and the gauge-kinetic couplings by performing an appropriate Kaluza-Klein reduction. We only discuss the couplings of the bulk moduli and leave their couplings to matter fields (and moduli) on the D-branes for a future investigation.²

In standard $N = 2$ Calabi-Yau compactifications the moduli space consists of two components, a special Kähler manifold \mathcal{M}^K and a quaternionic manifold \mathcal{M}^Q [15, 16, 17]. The orientifold projection truncates the $N = 2$ massless spectrum and thus defines a Kähler submanifold in the moduli space of Calabi-Yau compactifications [7, 18, 19, 20, 21, 22]. This submanifold continues to be a product of two components $\tilde{\mathcal{M}}^K \times \tilde{\mathcal{M}}^Q$. For type IIA orientifolds we show that $\tilde{\mathcal{M}}^K$ is again a special Kähler manifold characterized by a (truncated) holomorphic prepotential depending on the complexified Kähler deformations of the Calabi-Yau orientifold. The geometry of the Kähler submanifold $\tilde{\mathcal{M}}^Q$ inside \mathcal{M}^Q turns out to be more involved. This is due to the fact that the orientifold projection is anti-holomorphic and destroys the complex structure on the space of complex structure deformations. Instead the complex structure of $\tilde{\mathcal{M}}^Q$ combines the type IIA RR three-form C_3 with $\text{Re}\Omega$ to form a ‘new’ three-form $\Omega_c = C_3 + 2i\text{Re}(C\Omega)$ where C is related to the inverse dilaton. The Kähler coordinates of $\tilde{\mathcal{M}}^Q$ turn out to be half of the periods of Ω_c and the resulting geometry is similar to the geometry of the moduli space of Lagrangian submanifold as discussed in ref. [23]. The Kähler potential of $\tilde{\mathcal{M}}^Q$ encodes the dynamics of $\text{Re}(C\Omega)$.

Once background fluxes are turned on a superpotential W is generated. Also W decomposes into the sum of two terms analogously to the split of the Kähler geometry.

²We do include D-branes for consistency but we freeze their moduli and matter fields such that they do not appear in the low energy effective action.

As a consequence W depends on all geometric moduli of the Calabi-Yau orientifold. For $N = 2$ type IIA compactification this has also been observed recently in ref. [24]. The flux-induced superpotential receives further corrections from worldsheet and D -brane instantons. We do not compute such corrections here but observe that the chiral coordinates of $\tilde{\mathcal{M}}^Q$ are precisely such that they linearize the $D2$ -brane instanton action and hence holomorphic corrections to the superpotential are possible.

The type IIA orientifold compactifications considered in this paper are closely related to M-theory compactifications on a special class of G_2 manifolds [25, 26]. We show that the effective action of G_2 compactifications determined in refs. [27, 25, 28, 29, 30, 31, 32] indeed reduces in an appropriate limit to the effective action computed in this paper. This gives an alternative view on the geometry of $\tilde{\mathcal{M}}^Q$ since it can also be understood as a certain limit of the G_2 moduli space. In particular, the definition of Ω_c appear very naturally from an M-theory perspective.

Type IIA and type IIB compactification on Calabi-Yau threefolds are equivalent as a consequence of mirror symmetry [33]. In terms of the low energy effective action this implies that the two holomorphic $N = 2$ prepotentials of the special Kähler manifold \mathcal{M}^K and the quaternionic manifold \mathcal{M}^Q are equal. In this way mirror symmetry computes the worldsheet instanton corrections geometrically. One expects mirror symmetry to be also present in the orientifold versions of such compactifications [8, 10]. However, in this case the corrections are more difficult to control since one can only rely on $N = 1$ supersymmetry. Furthermore, since $\tilde{\mathcal{M}}^Q$ is not a special Kähler manifold its geometry is no longer encoded in a holomorphic function and hence determining the corrections is less straightforward. We take a purely supergravity point of view and compare the effective action computed in this paper with the type IIB mirror actions determined in [21]. Within this framework we find that for $\tilde{\mathcal{M}}^K$ mirror symmetry acts exactly as in $N = 2$ and equates the two orientifold truncated holomorphic prepotentials. For $\tilde{\mathcal{M}}^Q$ the situation is considerably more involved and depending on the orientifold projection two inequivalent mirrors do appear. On the type IIA side they correspond to two possible sets of special coordinates while in type IIB they give rise to $O3/O7$ planes or $O5/O9$ planes.

The paper is organized as follows. To set the stage we briefly review the compactification of type IIA on a Calabi-Yau manifold in section 2. In section 3 we turn to the discussion of type IIA Calabi-Yau orientifolds. The orientifold projection is introduced in section 3.1 and the resulting four-dimensional $N = 1$ spectrum is determined. In section 3.2 we calculate the effective action by performing a Kaluza-Klein reduction while additionally imposing the orientifold constraints. To bring the effective action in the standard $N = 1$ form we determine the Kähler coordinates, the Kähler potential and gauge-kinetic couplings in section 3.3.

In section 4 we redo the reduction starting from massive type IIA supergravity [34] while switching on the full set of possible NS and RR fluxes. This induces a superpotential for all geometric moduli which we determine explicitly. Furthermore we discuss contributions to the superpotential due to $D2$ instantons. By using the BPS conditions we show that the $D2$ instanton action becomes linear in the $N = 1$ coordinates, which in fact is a generic feature of all supersymmetric D-instantons in type II Calabi-Yau orientifolds.

The embedding of IIA Calabi-Yau orientifolds into an M-theory compactification on a special class of G_2 manifolds is discussed in section 5. We match explicitly the moduli spaces, gauge-couplings and parts of the flux superpotentials.

In section 6, we discuss mirror symmetry for Calabi-Yau orientifolds and determine the necessary conditions on the involutive symmetries of the mirror IIA and IIB orientifold theories. By specifying two types of special coordinates on the IIA side, we are able to identify the large complex structure limit of IIA orientifolds with the large volume limits of IIB orientifolds with $O3/O7$ and $O5/O9$ planes.

Section 7 contains our conclusions and some technical aspects of our analysis are presented in four appendices. Appendix A briefly reviews the special geometry of the Calabi-Yau moduli space. In appendix B we summarize $N = 1$ supergravities with several linear multiplets as they are relevant in the computation of the effective action. Appendix C contains the details of the reduction of the quaternionic manifold \mathcal{M}^Q for an arbitrary symplectic basis of H^3 . Finally, appendix D relates the geometry of $\tilde{\mathcal{M}}^Q$ to the moduli space of supersymmetric Lagrangian submanifolds of Calabi-Yaus following [23].

2 Type IIA compactified on Calabi-Yau threefolds

In order to set the stage for the orientifold compactifications we briefly review the compactification of type IIA supergravity on a Calabi-Yau threefold Y in this section [35]. Since our main concern is the $N = 2$ geometry of the moduli space we do not review the effective action where in addition background fluxes are turned on [36, 24].

We start from the ten-dimensional type IIA supergravity action in the Einstein frame given by

$$\begin{aligned} S_{IIA}^{(10)} = & \int -\frac{1}{2}\hat{R} * \mathbf{1} - \frac{1}{4}d\hat{\phi} \wedge *d\hat{\phi} - \frac{1}{4}e^{-\hat{\phi}}\hat{H}_3 \wedge *\hat{H}_3 - \frac{1}{2}e^{\frac{3}{2}\hat{\phi}}\hat{F}_2 \wedge *\hat{F}_2 \\ & - \frac{1}{2}e^{\frac{1}{2}\hat{\phi}}\hat{F}_4 \wedge *\hat{F}_4 + \mathcal{L}_{\text{top}} , \end{aligned} \quad (2.1)$$

where

$$\mathcal{L}_{\text{top}} = -\frac{1}{2}\left[\hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 - (\hat{B}_2)^2 \wedge d\hat{C}_3 \wedge d\hat{A}_1\right] , \quad (2.2)$$

and the field strengths are defined as

$$\hat{H}_3 = d\hat{B}_2 , \quad \hat{F}_2 = d\hat{A}_1 , \quad \hat{F}_4 = d\hat{C}_3 - \hat{A}_1 \wedge \hat{H}_3 . \quad (2.3)$$

The dilaton $\hat{\phi}$, the ten-dimensional metric \hat{g} and the two-form \hat{B}_2 are the massless fields in the NS sector, while the one- and three-forms \hat{A}_1, \hat{C}_3 arise in the RR sector.³

By compactifying this theory on a Calabi-Yau threefold Y one obtains an $N = 2$ theory in four space-time dimensions ($D = 4$) where the zero modes of Y assemble into massless $N = 2$ multiplets. These zero modes are in one-to-one correspondence with harmonic forms on Y and thus their multiplicity is counted by the dimension of the

³We use a ‘hat’ to denote ten-dimensional quantities and omit it for four-dimensional fields.

non-trivial cohomologies $H^{(1,1)}$ and $H^{(1,2)}$. More precisely, one takes the ten-dimensional metric to be block diagonal

$$ds^2 = \eta_{\mu\nu}(x) dx^\mu dx^\nu + g_{i\bar{j}}(y) dy^i dy^{\bar{j}} , \quad (2.4)$$

where $\eta_{\mu\nu}, \mu, \nu = 0, \dots, 3$ is a four-dimensional Minkowski metric and $g_{i\bar{j}}, i, \bar{j} = 1 \dots 3$ is a Calabi-Yau metric. Accordingly we expand the ten-dimensional gauge-potentials introduced in (2.3) in terms of harmonic forms on Y

$$\begin{aligned} \hat{A}_1 &= A^0(x) , & \hat{B}_2 &= B_2(x) + b^A(x) \omega_A , & A &= 1, \dots, h^{(1,1)} , \\ \hat{C}_3 &= A^A(x) \wedge \omega_A + \xi^{\hat{K}}(x) \alpha_{\hat{K}} - \tilde{\xi}_{\hat{K}}(x) \beta^{\hat{K}} , & \hat{K} &= 0, \dots, h^{(2,1)} . \end{aligned} \quad (2.5)$$

Here $b^A, \xi^{\hat{K}}, \tilde{\xi}_{\hat{K}}$ are four-dimensional scalars, A^0, A^A are one-forms and B_2 is a two-form. The harmonic forms ω_A form a basis of $H^{(1,1)}(Y)$ on the internal Calabi-Yau Y while the $(\alpha_{\hat{K}}, \beta^{\hat{K}})$ form a real symplectic basis of $H^3(Y)$ in that they satisfy

$$\int \alpha_{\hat{K}} \wedge \beta^{\hat{L}} = \delta_{\hat{K}}^{\hat{L}} , \quad (2.6)$$

with all other intersections vanishing. The ten-dimensional one-form \hat{A}_1 only contains a four-dimensional one-form A^0 in the expansion (2.5) since a Calabi-Yau threefold has no harmonic one-forms.

The four-dimensional massless modes are completed by also taking deformations of the Calabi-Yau metric $g_{i\bar{j}}$ into account. These deformations are divided into the deformations of the Kähler form J and deformations of the complex structure. The former correspond to $h^{(1,1)}$ real scalars v^A while the later are $h^{(1,2)}$ complex scalars $z^K, K = 1, \dots, h^{(2,1)}$.⁴ Together with the fields defined in the expansion (2.5) they assemble into a gravity multiplet $(g_{\mu\nu}, A^0)$, $h^{(1,1)}$ vector multiplets (A^A, v^A, b^A) , $h^{(2,1)}$ hypermultiplets $(z^K, \xi^K, \tilde{\xi}_K)$ and one tensor multiplet $(B_2, \phi, \xi^0, \tilde{\xi}_0)$ where we only give the bosonic components. Dualizing the two-form B_2 to a scalar a results in one further hypermultiplet. We summarize the bosonic spectrum in table 2.1.

gravity multiplet	1	$(g_{\mu\nu}, A^0)$
vector multiplets	$h^{(1,1)}$	(A^A, v^A, b^A)
hypermultiplets	$h^{(2,1)}$	$(z^K, \xi^K, \tilde{\xi}_K)$
tensor multiplet	1	$(B_2, \phi, \xi^0, \tilde{\xi}_0)$

Table 2.1: $N = 2$ multiplets for Type IIA supergravity compactified on a Calabi-Yau manifold.

In order to display the low energy effective action in the standard $N = 2$ form one needs to redefine the field variables slightly. One combines the real scalars v^A, b^A into complex fields t^A and defines a four-dimensional dilaton D according to⁵

$$t^A = b^A + i v^A , \quad e^D = e^\phi (\mathcal{K}/6)^{-\frac{1}{2}} , \quad (2.7)$$

⁴Note that \hat{K} introduced in (2.5) takes one more value than K in that it includes the zero.

⁵The fields v^A are defined as the expansion coefficients of the Kähler form J in the string-frame $J = v^A \omega_A$ which is related to the Kähler form J_E in the Einstein-frame via $J = e^{\phi/2} J_E$.

where $\mathcal{K} = \int J \wedge J \wedge J = 6 \text{vol}(Y)$ is proportional to volume of Y in the string-frame. Inserting the field expansions (2.5) into (2.3), (2.1) and reducing the Riemann scalar R by including the complex and Kähler deformations one ends up with the four-dimensional $N = 2$ effective action [37, 35, 17]

$$\begin{aligned} S_{\text{IIA}}^{(4)} = & \int -\frac{1}{2}R * \mathbf{1} + \frac{1}{2}\text{Im}\mathcal{N}_{\hat{A}\hat{B}} F^{\hat{A}} \wedge *F^{\hat{B}} + \frac{1}{2}\text{Re}\mathcal{N}_{\hat{A}\hat{B}} F^{\hat{A}} \wedge F^{\hat{B}} \\ & - G_{AB} dt^A \wedge *dt^B - h_{uv} d\tilde{q}^u \wedge *d\tilde{q}^v , \end{aligned} \quad (2.8)$$

where $F^{\hat{A}} = dA^{\hat{A}}$.

Let us first discuss the couplings of the hypermultiplet sector which are encoded in the quaternionic metric h_{uv} . From the Kaluza-Klein reduction one obtains [17]

$$\begin{aligned} h_{uv} d\tilde{q}^u d\tilde{q}^v = & (dD)^2 + G_{K\bar{L}} dz^K d\bar{z}^L + \frac{1}{4}e^{4D} (da - (\tilde{\xi}_{\hat{K}} d\xi^{\hat{K}} - \xi^{\hat{K}} d\tilde{\xi}_{\hat{K}}))^2 \\ & - \frac{1}{2}e^{2D} (\text{Im } \mathcal{M})^{-1} {}^{\hat{K}\hat{L}} (d\tilde{\xi}_{\hat{K}} - \mathcal{M}_{\hat{K}\hat{N}} d\xi^{\hat{N}}) (d\tilde{\xi}_{\hat{L}} - \bar{\mathcal{M}}_{\hat{L}\hat{M}} d\xi^{\hat{M}}) . \end{aligned} \quad (2.9)$$

$G_{K\bar{L}}$ is the metric on the submanifold \mathcal{M}^{cs} spanned by the complex structure deformations z^K and given by [38, 16]

$$G_{K\bar{L}} = -\frac{\int_Y \chi_K \wedge \bar{\chi}_L}{\int_Y \Omega \wedge \bar{\Omega}} . \quad (2.10)$$

χ_K is a basis of $(2, 1)$ -forms related to the variation of the three-form Ω via Kodaira's formula

$$\chi_K(z) = \partial_{z^K} \Omega(z) + \Omega(z) \partial_{z^K} K^{\text{cs}} . \quad (2.11)$$

With the help of (2.11) one shows that $G_{K\bar{L}}$ is a special Kähler metric determined by the periods of Ω

$$G_{K\bar{L}} = \partial_{z^K} \partial_{\bar{z}^L} K^{\text{cs}} , \quad K^{\text{cs}} = -\ln \left[i \int_Y \Omega \wedge \bar{\Omega} \right] = -\ln i \left[\bar{Z}^{\hat{K}} \mathcal{F}_{\hat{K}} - Z^{\hat{K}} \bar{\mathcal{F}}_{\hat{K}} \right] , \quad (2.12)$$

where the holomorphic periods $Z^{\hat{K}}, \mathcal{F}_{\hat{K}}$ are defined as

$$Z^{\hat{K}}(z) = \int_Y \Omega(z) \wedge \beta^{\hat{K}} , \quad \mathcal{F}_{\hat{K}}(z) = \int_Y \Omega(z) \wedge \alpha_{\hat{K}} , \quad (2.13)$$

or in other words Ω enjoys the expansion

$$\Omega(z) = Z^{\hat{K}}(z) \alpha_{\hat{K}} - \mathcal{F}_{\hat{K}}(z) \beta^{\hat{K}} . \quad (2.14)$$

$\mathcal{F}_{\hat{K}}$ is the first derivative with respect to $Z^{\hat{K}}$ of a prepotential $\mathcal{F} = \frac{1}{2}Z^{\hat{K}}\mathcal{F}_{\hat{K}}$. (We briefly summarize the special geometry of the Calabi-Yau moduli space in appendix A.)

Ω is only defined up to complex rescalings by a holomorphic function $e^{-h(z)}$ which via (2.12) also changes the Kähler potential by a Kähler transformation

$$\Omega \rightarrow \Omega e^{-h(z)} , \quad K^{\text{cs}} \rightarrow K^{\text{cs}} + h + \bar{h} . \quad (2.15)$$

This symmetry renders one of the periods (conventionally denoted by Z^0) unphysical in that one can always choose to fix a Kähler gauge and set $Z^0 = 1$. The complex structure

deformations can thus be identified with the remaining $h^{(1,2)}$ periods Z^K by defining the special coordinates $z^K = \frac{Z^K}{Z^0}$.

The complex coupling matrix $\mathcal{M}_{\hat{K}\hat{L}}$ appearing in (2.9) depends on the complex structure deformations z^K and is defined as [39]

$$\begin{aligned} \int \alpha_{\hat{K}} \wedge * \alpha_{\hat{L}} &= -(\text{Im } \mathcal{M} + (\text{Re } \mathcal{M})(\text{Im } \mathcal{M})^{-1}(\text{Re } \mathcal{M}))_{\hat{K}\hat{L}} , \\ \int \beta^{\hat{K}} \wedge * \beta^{\hat{L}} &= -(\text{Im } \mathcal{M})^{-1}{}^{\hat{K}\hat{L}} , \\ \int \alpha_{\hat{K}} \wedge * \beta^{\hat{L}} &= -((\text{Re } \mathcal{M})(\text{Im } \mathcal{M})^{-1})_{\hat{K}}^{\hat{L}} . \end{aligned} \quad (2.16)$$

It can be calculated from the periods (2.13) by using equation (A.8). Thus in the hypermultiplet sector all couplings are determined by a holomorphic prepotential and such metrics have been called dual or special quaternionic [40, 17].

Now let us turn to the couplings of the vector multiplets in the action (2.8). The metric $G_{A\bar{B}}$ only depends on the t^A (or rather their imaginary parts) and is defined as [15, 16]

$$G_{AB} = \frac{3}{2\mathcal{K}} \int_Y \omega_A \wedge * \omega_B = -\frac{3}{2} \left(\frac{\mathcal{K}_{AB}}{\mathcal{K}} - \frac{3}{2} \frac{\mathcal{K}_A \mathcal{K}_B}{\mathcal{K}^2} \right) = -\partial_{t^A} \partial_{\bar{t}^B} \ln \frac{4}{3} \mathcal{K} . \quad (2.17)$$

We abbreviated the intersection numbers as follows

$$\begin{aligned} \mathcal{K}_{ABC} &= \int_Y \omega_A \wedge \omega_B \wedge \omega_C , & \mathcal{K}_{AB} &= \int_Y \omega_A \wedge \omega_B \wedge J = \mathcal{K}_{ABC} v^C , \\ \mathcal{K}_A &= \int_Y \omega_A \wedge J \wedge J = \mathcal{K}_{ABC} v^B v^C , & \mathcal{K} &= \int_Y J \wedge J \wedge J = \mathcal{K}_{ABC} v^A v^B v^C , \end{aligned} \quad (2.18)$$

with $J = v^A \omega_A$ being the Kähler form of Y in the string-frame. The metric (2.17) is again a special Kähler metric in that the Kähler potential $K^K = -\ln \frac{4}{3} \mathcal{K}$ is also determined by a prepotential $f(t)$ given in (A.11) via (A.10).

Finally, the gauge-kinetic coupling matrix $\mathcal{N}_{\hat{A}\hat{B}}$ also depends on the scalars t^A and is given explicitly in (A.12). It can be calculated from a holomorphic prepotential as explained in appendix A.

As we have just reviewed the $N = 2$ moduli space has the local product structure

$$\mathcal{M}^K \times \mathcal{M}^Q , \quad (2.19)$$

where \mathcal{M}^K is the special Kähler manifold spanned by the scalars in the vector multiplets or in other words the (complexified) deformations of the Calabi-Yau Kähler form and \mathcal{M}^Q is a dual quaternionic manifold spanned by the scalars in the hypermultiplets. \mathcal{M}^Q has a special Kähler submanifold spanned by the complex structure deformations or in other words the geometric Calabi-Yau moduli space has the structure

$$\mathcal{M}^K \times \mathcal{M}^{cs} , \quad (2.20)$$

where both factors are special Kähler manifolds of complex dimension $h^{2,1}$ and $h^{1,1}$ respectively.

This ends our short review of Calabi-Yau compactifications of type IIA supergravity. Next we turn to its orientifold version which breaks $N = 2$ to $N = 1$ and as a consequence truncates the massless spectrum. This defines a Kähler submanifold inside the $N = 2$ moduli space (2.19). After determining the $N = 1$ spectrum we are going to find this Kähler subspace.

3 IIA orientifolds

After this brief review let us now turn to the main point of our paper and compactify type IIA supergravity on Calabi-Yau orientifolds. We first discuss the orientifold projection and the resulting $N = 1$ spectrum in section 3.1. In 3.2 we derive the effective action from a Kaluza-Klein reduction or equivalently by truncating the $N = 2$ action of the previous section. In 3.3 we find the appropriate chiral field variables which puts the action into the standard $N = 1$ form and determine the Kähler potential and the gauge kinetic function. In section 3.4 we redo the Kaluza-Klein reduction using as the starting point the massive ten-dimensional IIA supergravity of ref. [34]. We turn on background fluxes and determine the flux-induced superpotential. We also include a brief discussion of possible instanton corrections to the superpotential. Specifically we show that the $D2$ instanton action becomes linear in the chiral $N = 1$ coordinates and therefore holomorphic corrections to the superpotential can be induced.

3.1 The orientifold projection and the $N = 1$ spectrum

A Calabi-Yau orientifold is constructed from a Calabi-Yau manifold by modding out a discrete symmetry \mathcal{O} which includes the world-sheet parity Ω_p combined with the space-time fermion number in the left-moving sector $(-1)^{F_L}$. In addition \mathcal{O} can act non-trivially on the Calabi-Yau manifold so that one has altogether

$$\mathcal{O} = \Omega_p(-1)^{F_L}\sigma, \quad (3.1)$$

where σ is an involutive symmetry of Y (i.e. $\sigma^2 = 1$), acting trivially on the four flat dimensions. If one insists on preserving $N = 1$ supersymmetry σ has to be anti-holomorphic and isometric such that the Kähler form transforms as [8, 9, 10]

$$\sigma^*J = -J, \quad (3.2)$$

where σ^* denotes the pullback of the map σ . Compatibility of σ with the Calabi-Yau condition $\Omega \wedge \bar{\Omega} \propto J \wedge J \wedge J$ implies that σ also acts non-trivially on the three-form Ω as

$$\sigma^*\Omega = e^{2i\theta}\bar{\Omega}, \quad (3.3)$$

where $e^{2i\theta}$ is a constant phase and we included a factor 2 for later convenience.

Type IIA orientifolds with anti-holomorphic involution generically admit $O6$ planes. This is due to the fact, that the fixed point set of σ in Y are three-cycles Λ_n supporting the internal part of the orientifold planes. These cycles are special Lagrangian submanifolds of Y as an immediate consequence of (3.2) and (3.3) which implies [41]

$$J|_{\Lambda_n} = 0, \quad \text{Im}(e^{-i\theta}\Omega)|_{\Lambda_n} = 0. \quad (3.4)$$

In other words, they are calibrated with respect to $\text{Re}(e^{-i\theta}\Omega)$

$$\text{vol}(\Lambda_n) \sim \int_{\Lambda_n} \text{Re}(e^{-i\theta}\Omega) , \quad (3.5)$$

where the overall normalization of Ω will be determined in (4.18).⁶

In order to determine the \mathcal{O} -invariant states let us recall that the ten-dimensional RR forms \hat{A}_1 and \hat{C}_3 are odd under $(-1)^{F_L}$ while all other fields are even. Under the worldsheet parity Ω_p on the other hand \hat{B}_2, \hat{C}_3 are odd with all other fields being even. As a consequence the \mathcal{O} -invariant states have to satisfy [10]

$$\begin{aligned} \sigma^* \hat{\phi} &= \hat{\phi} , \\ \sigma^* \hat{g} &= \hat{g} , \\ \sigma^* \hat{B}_2 &= -\hat{B}_2 , \end{aligned} \quad \begin{aligned} \sigma^* \hat{A}_1 &= -\hat{A}_1 , \\ \sigma^* \hat{C}_3 &= \hat{C}_3 , \end{aligned} \quad (3.6)$$

while the deformations of the Calabi-Yau metric are constrained by (3.2) and (3.3).⁷

As we recalled in the previous section the massless modes are in one-to-one correspondence with the harmonic forms on Y . The space of harmonic forms splits under the involution σ into even and odd eigenspaces

$$H^p(Y) = H_+^p \oplus H_-^p . \quad (3.7)$$

Depending on the transformation properties given in (3.6) the \mathcal{O} -invariant states reside either in H_+^p or in H_-^p and as a consequence the number of states is reduced. We summarize all non-trivial cohomology groups including their basis elements in table 3.1.

cohomology group	$H_+^{(1,1)}$	$H_-^{(1,1)}$	$H_+^{(2,2)}$	$H_-^{(2,2)}$	$H_+^{(3)}$	$H_-^{(3)}$
dimension	$h_+^{(1,1)}$	$h_-^{(1,1)}$	$h_-^{(1,1)}$	$h_+^{(1,1)}$	$h^{(2,1)} + 1$	$h^{(2,1)} + 1$
basis	ω_α	ω_a	$\tilde{\omega}^a$	$\tilde{\omega}^\alpha$	$a_{\hat{K}}$	$b^{\hat{K}}$

Table 3.1: *Cohomology groups and their basis elements.*

ω_α, ω_a denote even and odd $(1, 1)$ -forms while $\tilde{\omega}^\alpha, \tilde{\omega}^a$ denote odd and even $(2, 2)$ -forms. The number of even $(1, 1)$ -forms is equal to the number of odd $(2, 2)$ -forms and vice versa since the volume form which is proportional to $J \wedge J \wedge J$ is odd and thus Hodge duality demands $h_+^{(1,1)} = h_-^{(2,2)}$, $h_-^{(1,1)} = h_+^{(2,2)}$. This can also be seen from the fact that the non-trivial intersection numbers are

$$\int \omega_\alpha \wedge \tilde{\omega}^\beta = \delta_\alpha^\beta , \quad \alpha, \beta = 1, \dots, h_+^{(1,1)} , \quad \int \omega_a \wedge \tilde{\omega}^b = \delta_a^b , \quad a, b = 1, \dots, h_-^{(1,1)} , \quad (3.8)$$

⁶As we discuss in section 4 this calibration condition plays a central role when including corrections due to BPS $D2$ instantons.

⁷Following the argument presented in [10] we note that the involution does not change under deformations of Y . This is due to its involutive property and the fact that we identify involutions which differ by diffeomorphisms. Therefore we fix an involution and restrict the deformation space by demanding (3.2) and (3.3).

with all other pairings vanishing. From the volume-form being odd one further infers $h_+^{(3,3)} = 0$, $h_-^{(3,3)} = 1$ and $h_+^{(0,0)} = 1$, $h_-^{(0,0)} = 0$.

H^3 can be decomposed independently of the complex structure as $H^3 = H_+^3 \oplus H_-^3$ where the (real) dimensions of both H_+^3 and H_-^3 is equal and given by $h_+^3 = h_-^3 = h^{(2,1)} + 1$. Again this is a consequence of Hodge duality together with the fact that the volume-form is odd. It implies that for each element $a_{\hat{K}} \in H_+^3$ there is a dual element $b^{\hat{L}} \in H_-^3$ with the intersections

$$\int a_{\hat{K}} \wedge b^{\hat{L}} = \delta_{\hat{K}}^{\hat{L}} , \quad \hat{K}, \hat{L} = 0, \dots, h^{(2,1)} . \quad (3.9)$$

Compared to (2.6) this amounts to a symplectic rotation such that all α -elements are chosen to be even and all β -elements are chosen to be odd but with the intersection numbers unchanged. The orientifold projection breaks this symplectic invariance or in other words fixes a particular symplectic gauge which groups all basis elements into even and odd. This in turn implies that the basis $(a_{\hat{K}}, b^{\hat{K}})$ is only one possible choice. However, since the calculation simplifies considerably for this basis, we first restrict to this special case and later give the general results with calculations summarized in appendix C.

In the remainder of this subsection we determine the $N = 1$ spectrum which survives the orientifold projections. Let us first discuss the Kähler moduli. From the eqs. (3.2) and (3.6) we see that both J and \hat{B}_2 are odd and hence have to be expanded in a basis ω_a of odd harmonic $(1, 1)$ -forms

$$J = v^a(x) \omega_a , \quad \hat{B}_2 = b^a(x) \omega_a , \quad a = 1, \dots, h_-^{(1,1)} . \quad (3.10)$$

In contrast to (2.5) the four-dimensional two-form B_2 gets projected out due to (3.6) and the fact that σ acts trivially on the flat dimensions. v^a and b^a are space-time scalars and as in $N = 2$ they can be combined into complex coordinates

$$t^a = b^a + i v^a , \quad J_c = B_2 + i J , \quad (3.11)$$

where we have also introduced the complexified Kähler form J_c . We see that in terms of the field variables the same complex structure is chosen as in $N = 2$ but the dimension of the Kähler moduli space is truncated from $h^{(1,1)}$ to $h_-^{(1,1)}$.

The number of complex structure deformations is similarly reduced since (3.3) constrains the possible deformations. To see this one performs a symplectic rotation on (2.14) and expands Ω in the basis of $H_+^p \oplus H_-^p$, i.e. as⁸

$$\Omega(z) = Z^{\hat{K}}(z) a_{\hat{K}} - \mathcal{F}_{\hat{L}}(z) b^{\hat{L}} . \quad (3.12)$$

Inserted into (3.3) one finds

$$\text{Im}(e^{-i\theta} Z^{\hat{K}}) = 0 , \quad \text{Re}(e^{-i\theta} \mathcal{F}_{\hat{K}}) = 0 . \quad (3.13)$$

The first set of equations are $h^{(2,1)} + 1$ real conditions for $h^{(2,1)}$ complex scalars z^K . One of these equations is redundant due to the scale invariance (2.15) of Ω . More precisely, the phase of e^{-h} can be used to trivially satisfy $\text{Im}(e^{-i\theta} Z^{\hat{K}}) = 0$ for one of the $Z^{\hat{K}}$. Thus

⁸Let us stress that at this point all $N = 2$ relations are still intact since (3.12) is just a specific choice of the standard $N = 2$ basis (2.14).

$\text{Im}(e^{-i\theta} Z^{\hat{K}}) = 0$ projects out $h^{(2,1)}$ real scalars, i.e. half of the complex structure deformations. Furthermore, in section 3.2 we will see the remaining real complex structure deformations span a Lagrangian submanifold $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$ with respect to the Kähler form inside \mathcal{M}^{cs} . Note that the second set of equations in (3.13) $\text{Re}(e^{-i\theta} \mathcal{F}_{\hat{K}}) = 0$ should not be read as equations determining the $z^{\hat{K}}$ but is a constraint on the periods (or equivalently the Yukawa couplings) of the Calabi-Yau which has to be fulfilled in order to admit an involutive symmetry with the property (3.3).⁹

As we have just discussed the complex rescaling (2.15) is reduced to the freedom of a real rescaling by (3.3). Under these transformations Ω and the Kähler potential K^{cs} change as

$$\Omega \rightarrow \Omega e^{-\text{Re}(h)}, \quad K^{\text{cs}} \rightarrow K^{\text{cs}} + 2\text{Re}(h), \quad (3.14)$$

when restricted to $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$. This freedom can be used to set one of the $\text{Re}(e^{-i\theta} Z^{\hat{K}})$ equal to one and tells us that Ω depends only on $h^{(2,1)}$ real deformation parameters. However, it will turn out to be more convenient to leave this gauge freedom intact and define a complex ‘compensator’ $C = re^{-i\theta}$ with the transformation property $C \rightarrow Ce^{\text{Re}(h)}$.¹⁰ Later on we will relate r to the inverse of the four-dimensional dilaton so that the scale invariant function $C\Omega$ depends on $h^{(2,1)} + 1$ real parameters. Using (3.12) $C\Omega$ enjoys the expansion

$$C\Omega = \text{Re}(CZ^{\hat{K}}) a_{\hat{K}} - i\text{Im}(C\mathcal{F}_{\hat{L}}) b^{\hat{L}}. \quad (3.15)$$

We are left with the expansion of the ten-dimensional fields \hat{A}_1 and \hat{C}_3 into harmonic forms. From (3.6) we learn that \hat{A}_1 is odd and so together with the fact that Y posses no harmonic one-forms and σ acts trivially on the flat dimensions the entire \hat{A}_1 is projected out. This corresponds to the fact that the $N = 2$ graviphoton A^0 is removed from the gravity multiplet, which in $N = 1$ only consists of the metric $g_{\mu\nu}$ as bosonic component. Finally, \hat{C}_3 is even and thus can be expanded according to

$$\hat{C}_3 = c_3(x) + A^\alpha(x) \wedge \omega_\alpha + C_3, \quad C_3 \equiv \xi^{\hat{K}}(x) a_{\hat{K}}, \quad (3.16)$$

where $\xi^{\hat{K}}$ are $h^{(2,1)} + 1$ real scalars, A^α are $h_+^{(1,1)}$ one-forms and c_3 is a three-form in four dimensions. c_3 contains no physical degree of freedom but as we will see in section 4 corresponds to a constant flux parameter in the superpotential. The real scalars $\xi^{\hat{K}}$ have to combine with the $h^{(2,1)}$ real complex structure deformations and the dilaton to form chiral multiplets. In the next section we will find that the appropriate complex fields arise from the combination

$$\Omega_c = C_3 + 2i\text{Re}(C\Omega). \quad (3.17)$$

Expanding Ω_c in a basis (3.9) of $H_+^3(Y)$ and using (3.15) and (3.16) we have

$$\Omega_c = 2N^{\hat{K}} a_{\hat{K}}, \quad N^{\hat{K}} = \frac{1}{2} \int \Omega_c \wedge \beta^{\hat{K}} = \frac{1}{2} (\xi^{\hat{K}} + 2i\text{Re}(CZ^{\hat{K}})). \quad (3.18)$$

Due to the orientifold projection the two three-forms Ω and C_3 each lost half of their degrees of freedom and combined into a new complex three-form Ω_c . As we will show

⁹This can also be seen as conditions arising in consistent truncations of $N = 2$ to $N = 1$ theories as discussed in ref. [18].

¹⁰This is reminiscent of the situation encountered in the computation of the entropy of $N = 2$ black holes where it is also convenient to leave this scale invariance intact [42].

in more detail in the next section the ‘good’ chiral coordinates in the $N = 1$ orientifold are the periods of $C\Omega$ directly while in $N = 2$ the periods agree with the proper field variables only in special coordinates.

Let us summarize the resulting $N = 1$ spectrum. It assembles into a gravitational multiplet, $h_+^{(1,1)}$ vector multiplets and $(h_-^{(1,1)} + h^{(2,1)} + 1)$ chiral multiplets. We list the bosonic parts of the $N = 1$ supermultiplets in table 3.2 [10]. We see that the $h^{(1,1)}$ $N = 2$ vector multiplets split into $h_+^{(1,1)}$ $N = 1$ vector multiplets and $h_-^{(1,1)}$ chiral multiplets while the $h^{(2,1)} + 1$ hypermultiplets are reduced to $h^{(2,1)} + 1$ chiral multiplets.

multiplets	multiplicity	bosonic components
gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h_+^{(1,1)}$	A^α
chiral multiplets	$h_-^{(1,1)}$	t^a
chiral multiplets	$h^{(2,1)} + 1$	$N^{\hat{K}}$

Table 3.2: $N = 1$ spectrum of orientifold compactification.

3.2 The effective action

In this section we calculate the four-dimensional effective action of type IIA orientifolds by performing a Kaluza-Klein reduction of the ten-dimensional type IIA action (2.1) taking the orientifold constraints into account. Equivalently this amounts to imposing the orientifold projections on the $N = 2$ action of section 2. Inserting (3.10), (3.15), (3.16) into the ten-dimensional type IIA action (2.1) and performing a Weyl rescaling of the four-dimensional metric we find

$$\begin{aligned} S_{O6}^{(4)} = & \int -\frac{1}{2}R * \mathbf{1} - G_{ab} dt^a \wedge *dt^b + \frac{1}{2}\text{Im} \mathcal{N}_{\alpha\beta} F^\alpha \wedge *F^\beta + \frac{1}{2}\text{Re} \mathcal{N}_{\alpha\beta} F^\alpha \wedge F^\beta \\ & - dD \wedge *dD - G_{KL}(q) dq^K \wedge *dq^L + \frac{1}{2}e^{2D} \text{Im} \mathcal{M}_{\hat{K}\hat{L}} d\xi^{\hat{K}} \wedge *d\xi^{\hat{L}}, \end{aligned} \quad (3.19)$$

where $F^\alpha = dA^\alpha$. Let us discuss the different couplings appearing in (3.19) in turn. Apart from the standard Einstein-Hilbert term the first line arises from the projection of the $N = 2$ vector multiplets action. As we already observed the orientifold projection reduces the number of Kähler moduli from $h^{(1,1)}$ to $h_-^{(1,1)}$ ($t^A \rightarrow t^a$) but leaves the complex structure on this component of the moduli space intact. Accordingly the metric $G_{ab}(t)$ is inherited from the metric G_{AB} of the $N = 2$ moduli space \mathcal{M}^{SK} given in (2.17). Since the volume form is odd only intersection numbers with one or three odd basis elements in (2.18) can be non-zero and consequently one has

$$\mathcal{K}_{\alpha\beta\gamma} = \mathcal{K}_{\alpha ab} = \mathcal{K}_{\alpha a} = \mathcal{K}_\alpha = 0, \quad (3.20)$$

while all other intersection numbers can be non-vanishing.¹¹ This implies that the metric $G_{AB}(t^A)$ of (2.17) is block diagonal and obeys

$$G_{ab} = -\frac{3}{2} \left(\frac{\mathcal{K}_{ab}}{\mathcal{K}} - \frac{3}{2} \frac{\mathcal{K}_a \mathcal{K}_b}{\mathcal{K}^2} \right) , \quad G_{\alpha\beta} = -\frac{3}{2} \frac{\mathcal{K}_{\alpha\beta}}{\mathcal{K}} , \quad G_{\alpha b} = 0 , \quad (3.21)$$

where

$$\mathcal{K}_{ab} = \mathcal{K}_{abc} v^c , \quad \mathcal{K}_{\alpha\beta} = \mathcal{K}_{\alpha\beta a} v^a , \quad \mathcal{K}_a = \mathcal{K}_{abc} v^b v^c , \quad \mathcal{K} = \mathcal{K}_{abc} v^a v^b v^c . \quad (3.22)$$

The same consideration also truncates the $N = 2$ gauge-kinetic coupling matrix $\mathcal{N}_{\hat{A}\hat{B}}$ explicitly given in (A.12). Inserting (3.20) and (3.22) one arrives at

$$\text{Re}\mathcal{N}_{\alpha\beta} = -\mathcal{K}_{\alpha\beta a} b^a , \quad \text{Im}\mathcal{N}_{\alpha\beta} = \mathcal{K}_{\alpha\beta} , \quad \mathcal{N}_{a\alpha} = \mathcal{N}_{0\alpha} = 0 . \quad (3.23)$$

(The other non-vanishing matrix elements $\mathcal{N}_{\hat{a}\hat{b}}$ arise in the potential (4.8) once fluxes are turned on.)

Let us now discuss the terms in the second line of (3.19) arising from the reduction of the $N = 2$ hypermultiplet action which is determined by the quaternionic metric (2.9). D is the the four-dimensional dilaton defined in (2.7). The metric G_{KL} is inherited from the $N = 2$ Kähler metric $G_{KL}(z, \bar{z})$ given in (2.12) and thus is the induced metric on the submanifold $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$ defined by the constraint (3.3). More precisely, the complex structure deformations respecting (3.3) can be determined from (2.11) by considering infinitesimal variations of Ω

$$\Omega(z + \delta z) = \Omega(z) + \delta z^K (\partial_{z^K} \Omega)_z = \Omega(z) - \delta z^K (K_{z^K}^{\text{cs}} \Omega - \chi_K)_z . \quad (3.24)$$

Now we impose the condition that both $\Omega(z + \delta z)$ and $\Omega(z)$ satisfy (3.3). This implies locally

$$\delta z^K \partial_{z^K} K^{\text{cs}} = \delta \bar{z}^K \partial_{\bar{z}^K} K^{\text{cs}} , \quad \delta z^K \sigma^* \chi_K = e^{2i\theta} \delta \bar{z}^K \bar{\chi}_K , \quad (3.25)$$

where $\partial_{z^K} K^{\text{cs}}$ and χ_K are restricted to $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$. Using the fact that K^{cs} is a Kähler potential and therefore $\partial_{z^K} K^{\text{cs}} \neq 0$, we conclude from the first equation in (3.25) that for each δz^K either the real or imaginary part has to be zero. This is consistent with the observation of the previous section that coordinates of $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$ can be identified with the real or imaginary part of the complex structure deformations z^K . To simplify the notation we call these deformations collectively q^K and denote the embedding map by $\rho : \mathcal{M}_{\mathbb{R}}^{\text{cs}} \hookrightarrow \mathcal{M}^{\text{cs}}$. Locally this corresponds to

$$\rho : q^K = (q^s, q^\sigma) \mapsto z^K = (q^s, iq^\sigma) , \quad (3.26)$$

for some splitting $z^K = (z^s, z^\sigma)$. In other words, the local coordinates on $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$ are $\text{Re}z^s = q^s$ and $\text{Im}z^\sigma = q^\sigma$ while $\text{Im}z^s = 0 = \text{Re}z^\sigma$. Using the second equation in (3.25), the embedding map (3.26) and the expression (2.10) for the $N = 2$ metric $G_{K\bar{L}}$ we also deduce that the Kähler form vanishes when pulled back to $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$. In summary we have

$$\rho^*(G_{K\bar{L}} dz^K d\bar{z}^L) \equiv G_{KL}(q) dq^K dq^L , \quad \rho^*(iG_{K\bar{L}} dz^K \wedge d\bar{z}^L) = 0 . \quad (3.27)$$

The first equation defines the induced metric while the second equation implies that $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$ is a Lagrangian submanifold of \mathcal{M}^{cs} with respect to the Kähler-form.

¹¹From a supergravity point of view this has been discussed also in [18].

Finally, coming back to the action (3.19) the matrix $\mathcal{M}_{\hat{K}\hat{L}}$ is defined in analogy with (2.16) as

$$\begin{aligned} \int a_{\hat{K}} \wedge *a_{\hat{L}} &= -\text{Im } \mathcal{M}_{\hat{K}\hat{L}} , & \int a_{\hat{K}} \wedge *b^{\hat{L}} &= 0 , \\ \int b^{\hat{K}} \wedge *b^{\hat{L}} &= -(\text{Im } \mathcal{M})^{-1}{}^{\hat{K}\hat{L}} , \end{aligned} \quad (3.28)$$

where $\text{Im } \mathcal{M}_{\hat{K}\hat{L}}$ can be given explicitly in terms of the periods by inserting (3.13) into (A.8). This yields

$$\text{Im } \mathcal{M}_{\hat{K}\hat{L}} = -\text{Im } \mathcal{F}_{\hat{K}\hat{L}} + 2 \frac{(\text{Im } \mathcal{F})_{\hat{K}\hat{M}} \text{Re}(CZ^{\hat{M}})(\text{Im } \mathcal{F})_{\hat{L}\hat{N}} \text{Re}(CZ^{\hat{N}})}{\text{Re}(CZ^{\hat{N}})(\text{Im } \mathcal{F})_{\hat{N}\hat{M}} \text{Re}(CZ^{\hat{M}})} . \quad (3.29)$$

Similarly one obtains $\text{Re } \mathcal{M}_{\hat{K}\hat{L}} = 0$ consistent with (2.16) which corresponds to the vanishing of the second intersection in (3.28).

This ends our discussion of the effective action obtained by applying the orientifold projection. The next step is to rewrite the action (3.19) in the standard $N = 1$ supergravity form which we turn to now.

3.3 The effective action in the $N=1$ supergravity form

In $N = 1$ supergravity the action is expressed in terms of a Kähler potential K , a holomorphic superpotential W and the holomorphic gauge-kinetic coupling functions f as follows [43, 44]

$$S^{(4)} = - \int \frac{1}{2} R * \mathbf{1} + K_{I\bar{J}} dM^I \wedge *d\bar{M}^{\bar{J}} + \frac{1}{2} \text{Re } f_{\alpha\beta} F^\alpha \wedge *F^\beta + \frac{1}{2} \text{Im } f_{\alpha\beta} F^\alpha \wedge F^\beta + V * \mathbf{1} , \quad (3.30)$$

where

$$V = e^K (K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2) + \frac{1}{2} (\text{Re } f)^{-1}{}^{\alpha\beta} D_\alpha D_\beta . \quad (3.31)$$

Here the M^I collectively denote all complex scalars in the theory and $K_{I\bar{J}}$ is a Kähler metric satisfying $K_{I\bar{J}} = \partial_I \bar{\partial}_{\bar{J}} K$. The scalar potential is expressed in terms of the Kähler-covariant derivative $D_I W = \partial_I W + (\partial_I K)W$.

Comparing (3.19) with (3.30) using (3.23) and (3.11) we can immediately read off the gauge-kinetic coupling function $f_{\alpha\beta}$ to be

$$f_{\alpha\beta} = -i\bar{N}_{\alpha\beta} = i\mathcal{K}_{\alpha\beta a} t^a . \quad (3.32)$$

As required by $N = 1$ supersymmetry the $f_{\alpha\beta}$ are indeed holomorphic. Note that they are linear in the t^a moduli and do not depend on the complex structure and ξ -moduli.

From (3.19) we also immediately observe that the orientifold moduli space has the product structure

$$\tilde{\mathcal{M}}^K \times \tilde{\mathcal{M}}^Q . \quad (3.33)$$

The first factor $\tilde{\mathcal{M}}^K$ is a subspace of the $N = 2$ moduli space \mathcal{M}^K with dimension $h_-^{(1,1)}$ spanned by the complexified Kähler deformations t^a . The second factor $\tilde{\mathcal{M}}^Q$ is

a subspace of the quaternionic manifold \mathcal{M}^Q with dimension $h^{(2,1)} + 1$ spanned by the complex structure deformations q^K , the dilaton D and the scalars $\xi^{\hat{K}}$ arising from C_3 . Let us discuss both factors in turn.

As we already stressed earlier the metric G_{ab} of (3.19) defined in (3.21) is a trivial truncation of the $N = 2$ special Kähler metric (2.17) and therefore remains special Kähler. The Kähler potential is given by

$$K^K = -\ln \left[\frac{i}{6} \mathcal{K}_{abc} (t - \bar{t})^a (t - \bar{t})^b (t - \bar{t})^c \right] = -\ln \left[\frac{4}{3} \int_Y J \wedge J \wedge J \right], \quad (3.34)$$

where J is the Kähler form in the string frame. Moreover, K^K can be obtained from the prepotential $f(t) = -\frac{1}{6} \mathcal{K}_{abc} t^a t^b t^c$ by using equation (A.10). It is well known that K^K obeys the standard no-scale condition [45]

$$K_{t^a} K^{t^a \bar{t}^b} K_{\bar{t}^b} = 3. \quad (3.35)$$

The geometry of the second component $\tilde{\mathcal{M}}^Q$ in (3.33) is considerably more complicated. This is due to the fact that (3.18) defines a new complex structure on the field space. In the following we sketch the calculation of the Kähler potential for the basis $(a_{\hat{K}}, b^{\hat{K}})$ and only summarize the results for a generic symplectic basis. The details of this more involved calculation will be presented in appendix C.

To begin with, let us define the compensator C introduced in section 3.1 as

$$C = e^{-D-i\theta} e^{K^{\text{cs}}(q)/2}, \quad C \rightarrow C e^{\text{Re} h(q)}, \quad (3.36)$$

where K^{cs} is the Kähler potential defined in (2.12) restricted to the real subspace $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$. We also displayed the transformation behavior of C under real Kähler transformations (3.14). With this at hand one defines the scale invariant variable

$$l^{\hat{K}} = \text{Re}(C Z^{\hat{K}}(q)). \quad (3.37)$$

Inserted into (3.19) and using the Jacobian matrix encoding the change of variables $(e^D, q^K) \rightarrow l^{\hat{K}}$ the second line (3.19) simplifies as¹²

$$\mathcal{L}_Q^{(4)} = 2e^{2D} \text{Im} \mathcal{M}_{\hat{K}\hat{L}} (dl^{\hat{K}} \wedge *dl^{\hat{L}} + \frac{1}{4} d\xi^{\hat{K}} \wedge *d\xi^{\hat{L}}). \quad (3.38)$$

We see that the scalars $l^{\hat{K}}$ and $\xi^{\hat{K}}$ nicely combine into complex coordinates

$$N^{\hat{K}} = \frac{1}{2} \xi^{\hat{K}} + i l^{\hat{K}} = \frac{1}{2} \xi^{\hat{K}} + i \text{Re}(C Z^{\hat{K}}) = \frac{1}{2} \int \Omega_c \wedge b^{\hat{K}}, \quad (3.39)$$

which we anticipated in equation (3.18). The important fact to note here is that $\tilde{\mathcal{M}}^Q$ is equipped with a new complex structure and the corresponding Kähler coordinates coincide with half of the periods of Ω_c . This is in contrast to the situation in $N = 2$ where one of the periods (Z^0) is a gauge degree of freedom and the Kähler coordinates are the special coordinates $z^K = Z^K/Z^0$.

¹²The calculation of this result can be found in appendix C.

In order to show that the metric in (3.38) is Kähler we need the explicit expression for the Kähler potential. Using (3.29) one obtains straightforwardly

$$2e^{2D} \text{Im} \mathcal{M}_{\hat{K}\hat{L}} = \partial_{N^{\hat{K}}} \partial_{\bar{N}^{\hat{L}}} K^Q , \quad (3.40)$$

where

$$K^Q = -2 \ln [4i\mathcal{F}(CZ)] , \quad \mathcal{F}(\text{Re}(CZ)) = \frac{i}{2} \text{Re}(CZ^{\hat{K}}) \text{Im}(C\mathcal{F}_{\hat{K}}) . \quad (3.41)$$

Alternatively, using (3.15) and $*\Omega = -i\Omega$ one derives the integral representation

$$K^Q = -2 \ln \left[2 \int_Y \text{Re}(C\Omega) \wedge * \text{Re}(C\Omega) \right] = -\ln e^{-4D} , \quad (3.42)$$

where in the second equation we used (3.36) and (2.12). In the form (3.42) the dependence of K^Q on the coordinates $N^{\hat{K}}$ is only implicit and given by means of their definition (3.39). Also K^Q obeys a no-scale type condition in that it satisfies

$$K_{N^{\hat{K}}} K^{N^{\hat{K}} \bar{N}^{\hat{L}}} K_{\bar{N}^{\hat{L}}} = 4 , \quad (3.43)$$

which can be checked by direct calculation.

The analysis so far started from the symplectic basis $(a_{\hat{K}}, b^{\hat{K}})$ introduced in (3.9), determined the Kähler coordinates in (3.39) and derived the Kähler potential K^Q in terms of the prepotential \mathcal{F} in (3.41) or as an integral representation in (3.42). Now we need to ask to what extent this result depends on the choice of the basis (3.9). Or in other words let us redo the calculation starting from an arbitrary symplectic basis and determine the Kähler potential and the proper field variables for the corresponding orientifold theory. Let us first recall the situation in the $N = 2$ theory reviewed in section 2. The periods $(Z^{\hat{K}}, \mathcal{F}_{\hat{K}})$ defined in (2.13) form a symplectic vector of $Sp(2h^{(1,2)} + 2, \mathbf{Z})$ such that Ω given in (2.14) and K^{cs} given in (2.12) is manifestly invariant. The prepotential $\mathcal{F}(Z) = \frac{1}{2} Z^{\hat{K}} \mathcal{F}_{\hat{K}}$ on the other hand does depend on the choice of the basis $(\alpha_{\hat{K}}, \beta^{\hat{K}})$ and is not invariant.

For $N = 1$ orientifolds this situation is different since the orientifold projection (3.3) explicitly breaks the symplectic invariance.¹³ This can also be seen from the form of the $N = 1$ Kähler potential (3.41) which is expressed in terms of the non-invariant prepotential. One immediately concludes that the result (3.41) is basis dependent and K^Q takes this simple form due to the special choice $a_{\hat{K}} \in H_+^3(Y)$ and $b^{\hat{K}} \in H_-^3(Y)$.¹⁴ On the other hand, the integral representation (3.42) only implicitly depends on the symplectic basis through the definition of the coordinates $N^{\hat{K}}$. This suggest, that it is possible to generalize our results by allowing for an arbitrary choice of symplectic basis in the definition of the $N = 1$ coordinates. More precisely, let us consider the generic basis $(\alpha_{\hat{K}}, \beta^{\hat{L}})$, where we assume that the $h_+^3 = h^{2,1} + 1$ basis elements $(\alpha_k, \beta^\lambda)$ span H_+^3

¹³A symplectic transformation \mathcal{S} preserve the form $\langle \alpha, \beta \rangle = \int \alpha \wedge \beta$, such that $\langle \mathcal{S}\alpha, \mathcal{S}\beta \rangle = \langle \alpha, \beta \rangle$. On the other hand the anti-holomorphic involution satisfies $\langle \sigma^* \alpha, \sigma^* \beta \rangle = -\langle \alpha, \beta \rangle$.

¹⁴Note that this is in striking analogy to the background dependence of the B model partition function as discussed in [46, 47]

and the $h_{-}^3 = h^{2,1} + 1$ basis elements $(\alpha_{\lambda}, \beta^k)$ span H_{-}^3 . In this basis the intersections (2.6) take the form

$$\int_Y \alpha_k \wedge \beta^l = \delta_k^l, \quad \int_Y \alpha_{\lambda} \wedge \beta^{\lambda} = \delta_{\lambda}^{\lambda}, \quad (3.44)$$

with all other combinations vanishing. Applying the orientifold constraint (3.3) one concludes that the equations (3.13) are replaced by

$$\text{Im}(CZ^k) = \text{Re}(C\mathcal{F}_k) = 0, \quad \text{Re}(CZ^{\lambda}) = \text{Im}(C\mathcal{F}_{\lambda}) = 0. \quad (3.45)$$

Correspondingly, the expansions (3.15) and (3.16) take the form

$$\begin{aligned} C\Omega &= \text{Re}(CZ^k)\alpha_k + i\text{Im}(CZ^{\lambda})\alpha_{\lambda} - \text{Re}(C\mathcal{F}_{\lambda})\beta^{\lambda} - i\text{Im}(C\mathcal{F}_k)\beta^k, \\ C_3 &= \xi^k \alpha_k - \tilde{\xi}_{\lambda} \beta^{\lambda}, \end{aligned} \quad (3.46)$$

which implies that we also have to redefine the $N = 1$ coordinates of $\tilde{\mathcal{M}}^Q$ in an appropriate way. In appendix C we show that the new Kähler coordinates (N^k, T_{λ}) are again determined by the periods of Ω_c and given by

$$\begin{aligned} N^k &= \frac{1}{2} \int \Omega_c \wedge \beta^k = \frac{1}{2} \xi^k + i\text{Re}(CZ^k), \\ T_{\lambda} &= i \int \Omega_c \wedge \alpha_{\lambda} = i\tilde{\xi}_{\lambda} - 2\text{Re}(C\mathcal{F}_{\lambda}), \end{aligned} \quad (3.47)$$

where we evaluated the integrals by using (3.17) and (3.46).

The Kähler potential takes again the form (3.42) but now depends on N^k, T_{λ} and thus no longer simplifies to (3.41). Let us compare the situation to the original $N = 2$ theory, which was formulated in terms of the $Z^{\hat{K}}$ or equivalently the special coordinates z^K . Holomorphicity in these coordinates played a central role in defining the prepotential encoding the special geometry of \mathcal{M}^{cs} in \mathcal{M}^Q (cf. section 2). In contrast, the $N = 1$ orientifold constraints destroy this complex structure and force us to combine $\text{Re}(C\Omega)$ with the RR three-form C_3 into Ω_c . The Kähler coordinates are half of the periods of Ω_c but now in this more general case also the derivatives of \mathcal{F} can serve as coordinates as seen in (3.47). However, as it is shown in appendix C, $\text{Re}(C\mathcal{F}_{\lambda})$ and $e^{2D}\text{Im}(CZ^{\lambda})$ are related by a Legendre transformation of the Kähler potential. Working with this transformed potential and the coordinates $\text{Re}(CZ^k)$ and $e^{2D}\text{Im}(CZ^{\lambda})$ enables us to make contact to the underlying $N = 2$ theory in its canonical formulation. From a supergravity point of view, this Legendre transformation corresponds to replacing the chiral multiplets T_{λ} by linear multiplets as described in appendix B and C. This is possible due to the translational isometries of K , which arise as a consequence of the C_3 gauge invariance and which render K independent of the scalars ξ and $\tilde{\xi}$. We show in appendix D that this also enables us to construct $\tilde{\mathcal{M}}^Q$ from $\mathcal{M}_{\mathbb{R}}^{cs}$ similar to the moduli space of supersymmetric Lagrangian submanifolds in a Calabi-Yau space as described by Hitchin [23]. This also allows us to interpret the no-scale condition (3.43) geometrically.

Let us summarize the results obtained so far. We found that the moduli space of $N = 1$ orientifolds is indeed the product of two Kähler spaces with the Kähler potential

$$K = K^K + K^Q = -\ln \left[\frac{4}{3} \int_Y J \wedge J \wedge J \right] - 2 \ln \left[2 \int_Y \text{Re}(C\Omega) \wedge * \text{Re}(C\Omega) \right]. \quad (3.48)$$

The first term depends on the Kähler deformations of the orientifold while the second term is a function of the real complex structure deformations and the dilaton. The $N = 1$ Kähler coordinates are obtained by expanding the complex combinations¹⁵

$$\Omega_c = C_3 + 2i\text{Re}(C\Omega), \quad J_c = \hat{B}_2 + iJ, \quad (3.49)$$

in a real harmonic basis of $H_+^3(Y)$ and $H_-^{(1,1)}(Y)$ respectively. Note that K does not depend on the scalars arising in the expansion of \hat{B}_2 and \hat{C}_3 , such that the Kähler manifold admits a set of $h_-^{(1,1)} + h^{(2,1)} + 1$ translational isometries. In other words K consists of two functionals encoding the dynamics of the two-form J and the real three-form $\text{Re}(C\Omega)$.¹⁶ Moreover, irrespective of the chosen basis the Kähler potential obeys the no-scale type conditions (3.35) and (3.43), (C.20).

However, these two statements are violated when further stringy corrections are included. K receives additional contributions due to perturbative effects as well as world-sheet and $D2$ instantons. It is well-known that the combination $J_c = \hat{B}_2 + iJ$ gives the proper coupling to the string world-sheet such that world-sheet instantons correct the holomorphic prepotential as $f(t) = -\frac{1}{6}\mathcal{K}_{abc}t^a t^b t^c + O(e^{-t})$. Since we divided out the world-sheet parity these corrections also include non-orientable Riemann surfaces, such that the prepotential $f(t)$ consists of two parts $f(t) = f_{or}(t) + f_{unor}(t)$. The function f_{or} counts holomorphic maps from orientable world-sheets to Y , while f_{unor} counts holomorphic maps from non-orientable world-sheets to Y [49]. In the next section we show that $D2$ instantons naturally couple to the complex three-form Ω_c and they are expected to correct K^Q .

4 The effective action in the presence of background fluxes

In this section we derive the effective action of type IIA orientifolds in the presence of background fluxes. For standard $N = 2$ Calabi-Yau compactifications of type IIA a similar analysis is carried out in refs. [36, 24]. In order to do so we need to start from the ten-dimensional action of massive type IIA supergravity which differs from the action (2.1) in that the two-form \hat{B}_2 is massive. In the Einstein frame it is given by [34]

$$\begin{aligned} S_{MIIA}^{(10)} = & \int -\frac{1}{2}\hat{R} * \mathbf{1} - \frac{1}{4}d\hat{\phi} \wedge *d\hat{\phi} - \frac{1}{4}e^{-\hat{\phi}}\hat{H}_3 \wedge *\hat{H}_3 - \frac{1}{2}e^{\frac{3}{2}\hat{\phi}}\hat{F}_2 \wedge *\hat{F}_2 \\ & - \frac{1}{2}e^{\frac{1}{2}\hat{\phi}}\hat{F}_4 \wedge *\hat{F}_4 - \frac{1}{2}e^{\frac{5}{2}\hat{\phi}}(m^0)^2 * \mathbf{1} + \mathcal{L}_{\text{top}}, \end{aligned} \quad (4.1)$$

where

$$\begin{aligned} \mathcal{L}_{\text{top}} = & -\frac{1}{2}\left[\hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 - (\hat{B}_2)^2 \wedge d\hat{C}_3 \wedge d\hat{A}_1 + \frac{1}{3}(\hat{B}_2)^3 \wedge (d\hat{A}_1)^2 \right. \\ & \left. - \frac{m^0}{3}(\hat{B}_2)^3 \wedge d\hat{C}_3 + \frac{m^0}{4}(\hat{B}_2)^4 \wedge d\hat{A}_1 + \frac{(m^0)^2}{20}(\hat{B}_2)^5 \right], \end{aligned} \quad (4.2)$$

¹⁵This combination of forms has also appeared recently in ref. [48] in the discussion of D -instanton couplings in the A-model. Here they appear as the proper chiral $N = 1$ variables and as we will see in the next section they linearize the D-instanton action.

¹⁶The functions $V[\text{Re}(C\Omega)] = \int \text{Re}(C\Omega) \wedge *\text{Re}(C\Omega)$ and $V[J] = \int J \wedge J \wedge J$ are known as Hitchins functionals [30]. The orientifold constraints (3.2) and (3.3) restricts their domain to $J \in H_-^2(Y)$ and $\text{Re}(C\Omega) \in H_+^3(Y)$.

and the field strengths are defined as

$$\hat{H}_3 = d\hat{B}_2, \quad \hat{F}_2 = d\hat{A}_1 + m^0 \hat{B}_2, \quad \hat{F}_4 = d\hat{C}_3 - \hat{A}_1 \wedge \hat{H}_3 - \frac{m^0}{2} (\hat{B}_2)^2. \quad (4.3)$$

Compared to the analysis of the previous section we now include non-trivial background fluxes of the field strengths F_2 , H_3 and F_4 on the Calabi-Yau orientifold. We keep the Bianchi identity and the equation of motion intact and therefore expand F_2 , H_3 and F_4 in terms of harmonic forms compatible with the orientifold projection. From (3.6) we infer that F_2 is expanded in harmonic forms of $H_-^2(Y)$, H_3 in harmonic forms of $H_-^3(Y)$ and F_4 in harmonic forms of $H_+^4(Y)$.¹⁷ Explicitly the expansions read

$$H_3 = q^\lambda \alpha_\lambda - p_k \beta^k, \quad F_2 = -m^a \omega_a, \quad F_4 = e_a \tilde{\omega}^a, \quad (4.4)$$

where (q^λ, p_k) are $h^{(2,1)} + 1$ real NS flux parameters while (e_a, m^a) are $2h_-^{1,1}$ real RR flux parameters. The harmonic forms $(\alpha_\lambda, \beta^k)$ are the elements of the real symplectic basis of H_-^3 introduced in (3.44). The basis $\tilde{\omega}^a$ of $H_+^{(2,2)}$ is defined to be the dual basis of ω_a while the basis $\tilde{\omega}^\alpha$ denotes a basis of $H_-^{(2,2)}$ dual to ω_α .

Inserting (3.10), (3.16) and (4.4) into (4.3) we arrive at

$$\begin{aligned} \hat{H}_3 &= db^a \wedge \omega_a + q^\lambda \alpha_\lambda - p_k \beta^k, & \hat{F}_2 &= (m^0 b^a + m^a) \omega_a, \\ \hat{F}_4 &= dC_3 + dA^\alpha \wedge \omega_\alpha + d\xi^k \wedge \alpha_k - d\tilde{\xi}_\lambda \wedge \beta^\lambda + (b^a m^b - \frac{1}{2} m^0 b^a b^b) \mathcal{K}_{abc} \tilde{\omega}^c + e_a \tilde{\omega}^a, \end{aligned} \quad (4.5)$$

where we have used $\omega_a \wedge \omega_b = \mathcal{K}_{abc} \tilde{\omega}^c$. Now we repeat the KK-reduction of the previous section using the modified field strength (4.5) and the action (4.1) instead of (2.1). This results in¹⁸

$$S^{(4)} = S_{O6}^{(4)} - \int \frac{g}{2} dc_3 \wedge *dc_3 + h dc_3 + U * \mathbf{1}, \quad (4.6)$$

where $S_{O6}^{(4)}$ is given in (3.19). c_3 is the four-dimensional part of the ten-dimensional three-form \hat{C}_3 defined in (3.16) and its couplings to the scalar fields are given by

$$g = e^{-4\phi} \left(\frac{\mathcal{K}}{6}\right)^3, \quad h = e_a b^a + \tilde{\xi}_\lambda q^\lambda - \xi^k p_k + \frac{1}{2} \text{Re} \mathcal{N}_{0\hat{a}} m^{\hat{a}}, \quad (4.7)$$

where we denoted $m^{\hat{a}} = (m^0, m^a)$. The potential term U of (4.6) is given by

$$U = \frac{9}{\mathcal{K}^2} e^{2\phi} \int_Y H_3 \wedge *H_3 - \frac{18}{\mathcal{K}^2} e^{4\phi} \text{Im} \mathcal{N}_{\hat{a}\hat{b}} m^{\hat{a}} m^{\hat{b}} + \frac{27}{\mathcal{K}^3} e^{4\phi} G^{ab} (e_a - \text{Re} \mathcal{N}_{a\hat{a}} m^{\hat{a}}) (e_b - \text{Re} \mathcal{N}_{b\hat{b}} m^{\hat{b}}), \quad (4.8)$$

where

$$\int_Y H_3 \wedge *H_3 = -(p_k - \text{Re} \mathcal{M}_{k\lambda} q^\lambda) (\text{Im} \mathcal{M})^{-1}{}^{kl} (p_l - \text{Re} \mathcal{M}_{l\lambda} q^\lambda) - \text{Im} \mathcal{M}_{\kappa\lambda} q^\kappa q^\lambda. \quad (4.9)$$

The matrix $\mathcal{N}_{\hat{a}\hat{b}}(t, \bar{t})$ is defined to be the corresponding part of the $N = 2$ gauge-coupling matrix (A.12) restricted to $\tilde{\mathcal{M}}^K$ by applying (3.20) and (3.21). Similarly the matrices

¹⁷As we observed in the previous section there is no \hat{A}_1 due to the absence of one-forms on the orientifold. Nevertheless its field strength F_2 can be non-trivial on the orientifold since Y generically possesses non-vanishing harmonic two-forms.

¹⁸The action $S_{O6}^{(4)}$ is given in (3.19). However, due to the fact that we perform the Kaluza-Klein reduction in the generic basis introduced in (3.44) the kinetic terms for $\tilde{\mathcal{M}}^Q$ are replaced by (C.3).

$\mathcal{M}_{l\lambda}, \mathcal{M}_{\kappa\lambda}, \mathcal{M}_{kl}$ are obtained from the $N = 2$ matrix $\mathcal{M}_{\hat{K}\hat{L}}$ defined in (A.8) by applying the orientifold constraints (3.45), i.e. restricting them to the subspace $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$.

In four space-time dimensions c_3 is dual to constant which plays the role of an additional electric flux e_0 in complete analogy with the situation in $N = 2$ discussed in [36]. In order to write the action in terms of e_0 instead of c_3 we follow [36] and add it as a Lagrange multiplier to the action (4.6)

$$S^{(4)} \rightarrow S^{(4)} + e_0 dc_3 . \quad (4.10)$$

Treating dc_3 as an independent four-form its equation of motion reads $*dc_3 = -(h + e_0)/g$ which can be used to eliminate dc_3 in favor of e_0 .¹⁹ Inserted back into (4.10) one finds

$$S^{(4)} = S_{O6}^{(4)} + \int V * \mathbf{1} , \quad (4.11)$$

where

$$V = U + \int \frac{1}{2g} (h + e_0)^2 . \quad (4.12)$$

Inserting (4.8) we arrive at

$$V = \frac{9}{\kappa^2} e^{2\phi} \int H_3 \wedge *H_3 - \frac{18}{\kappa^2} e^{4\phi} (\tilde{e}_{\hat{a}} - \mathcal{N}_{\hat{a}\hat{c}} m^{\hat{c}}) (\text{Im} \mathcal{N})^{-1} {}^{\hat{a}\hat{b}} (\tilde{e}_{\hat{b}} - \bar{\mathcal{N}}_{\hat{b}\hat{c}} m^{\hat{c}}) , \quad (4.13)$$

where we introduced the shorthand notation $\tilde{e}_{\hat{a}} = (e_0 + \xi_{\lambda} q^{\lambda} - \xi^{\hat{k}} p_{\hat{k}}, e_a)$ and $m^{\hat{a}} = (m^0, m^a)$. Note that in the presence of NS flux one can absorb e_0 by shifting the fields $\xi, \tilde{\xi}$. This corresponds to adding an integral form to C_3 as carefully discussed in [32]. However, for the discussion of mirror symmetry it is more convenient to keep the parameter e_0 explicitly in the action.

In order to establish the consistency with $N = 1$ supergravity we need to rewrite V given in (4.13) in terms of (3.31) or in other words we need express V in terms of a superpotential W and appropriate D -terms. From (4.6) we infer that turning on fluxes does not charge any of the fields and therefore all D -terms have to vanish.²⁰ In order to determine W we first need to compute the inverse Kähler metric. Using (B.11), (C.10) and (2.16) we find

$$\begin{aligned} K^{T_{\kappa}\bar{T}_{\lambda}} &= 2e^{-2D} \int \alpha_{\kappa} \wedge * \alpha_{\lambda} , & K^{T_{\lambda}\bar{N}^k} &= ie^{-2D} \int \alpha_{\lambda} \wedge * \beta^k , \\ K^{N^k\bar{N}^l} &= \frac{1}{2} e^{-2D} \int \beta^k \wedge * \beta^l , & K^{t^a\bar{t}^a} &= G^{ab} . \end{aligned} \quad (4.14)$$

With the help of (4.14), (3.31) and (3.48) one checks that the potential (4.13) can be entirely expressed in terms of the superpotential

$$W = W^Q(N, T) + W^K(t) , \quad (4.15)$$

¹⁹An alternative derivation is given in ref. [32]. Minimizing U with respect to dc_3 sets it to the value $*dc_3 = -h/g$. Inserted back into U only gives its classical value while quantum mechanical states labeled by integers e_0 shift h as given in (4.12).

²⁰In type IIB orientifolds with $O5/O9$ planes a D-term and massive tensor fields appeared when NS-flux are turned on [21]. The mirror symmetric situation corresponds to compactifications on half-flat manifolds exactly as in $N = 2$ [50]. Work along these lines is in progress [51].

where

$$\begin{aligned}
W^Q(N^k, T_\lambda) &= \int_Y \Omega_c \wedge H_3 = -2N^k p_k - iT_\lambda q^\lambda, \\
W^K(t^a) &= e_0 + \int_Y J_c \wedge F_4 - \frac{1}{2} \int_Y J_c \wedge J_c \wedge F_2 - \frac{1}{6} m^0 \int_Y J_c \wedge J_c \wedge J_c, \\
&= e_0 + e_a t^a + \frac{1}{2} \mathcal{K}_{abc} m^a t^b t^c - \frac{1}{6} m^0 \mathcal{K}_{abc} t^a t^b t^c,
\end{aligned} \tag{4.16}$$

and Ω_c and J_c are defined in (3.49). We see that the superpotential is the sum of two terms. W^Q depends on the NS fluxes (p_k, q^λ) of H_3 and the chiral fields N^k, T_λ parameterizing the space $\tilde{\mathcal{M}}^Q$. W^K depends on the RR fluxes (e_a, m^b) of F_2 and F_4 (together with m^0 and e_0) and the complexified Kähler deformations t^a parameterizing \mathcal{M}^K . We see that contrary to the type IIB case both types of moduli, Kähler and complex structure deformations appear in the superpotential suggesting the possibility that all moduli can be fixed in this set-up. This has recently also been observed in ref. [24]. A more detailed phenomenological investigation will be presented elsewhere.

Let us close this section by briefly discussing possible instanton corrections to the superpotential (4.15). They can arise from worldsheet instantons wrapping the string around two-cycles of the orientifold or from wrapping $D2$ -branes around three-cycles Σ_3 [52]. The first set of corrections contribute analogously to the $N = 2$ theory with the difference that also non-oriented worldsheets can contribute as discussed at the end of the previous section.

The second set of correction comes from wrapping $D2$ -branes around three-cycles and can be viewed as the mirror symmetric corrections to the ones discussed in [53]. A computation of such corrections is beyond the scope of this paper but let us make the observation that they amount to holomorphic contribution in W when expressed in the proper Kähler variables (3.47). This can be seen from the fact that any correlation function is weighted by the string-frame world-volume action of the wrapped Euclidean $D2$ -branes and thus includes a factor $e^{-S_{D2}}$ where²¹

$$S_{D2} = -\mu_3 e^{-\hat{\phi}} \int_{\mathcal{W}_3} d^3 \lambda \sqrt{\det(\varphi^*(\hat{g} + \hat{B}_2) + 2\pi\alpha' F_2)} + i\mu_3 \int_{\mathcal{W}_3} \varphi^*(\hat{C}_3). \tag{4.17}$$

\mathcal{W}_3 is the world-volume of the $D2$ -brane and φ^* is the pullback of the map φ which embeds \mathcal{W}_3 into Calabi-Yau orientifold Y $\varphi : \mathcal{W}_3 \hookrightarrow Y$. The first term is the Dirac-Born-Infeld action describing the couplings of the $D2$ -brane to the bulk metric and the bulk \hat{B} -field while the second term is the Chern-Simons action which represents the coupling to the RR 3-form \hat{C}_3 . We have chosen the RR charge μ_3 equal to the tension since the wrapped $D2$ -branes must be BPS in order to preserve $N = 1$ supersymmetry. In fact there is an additional condition arising from the requirement that the $D2$ -branes preserves the same supersymmetry that is left intact by the orientifold projections. This in turn implies that both the $D2$ -brane and the internal part of the $O6$ -planes wrap special Lagrangian cycles calibrated with respect to the same real three-form.

The calibration condition for Euclidean $D2$ -branes has been derived in refs. [52, 54]. In order to adjust the normalization to the case at hand let us recall that the unbroken

²¹The possible extra term $\hat{A}_1 \wedge \hat{B}_2$ does not appear in the Chern-Simons part of (4.17) since \hat{A}_1 is projected out by the orientifold.

supercharge has to be some linear combination $\epsilon = a^+ \epsilon_+ + a^- \epsilon_-$ of the two covariantly constant spinors ϵ_+ and ϵ_- of the original $N = 2$ supersymmetry. Let us denote the relative phase of a^+ and a^- by $a^-/a^+ = -ie^{i\theta_B}$ while the absolute magnitude can be fixed by the normalization of Ω . From $\int J^3 = \frac{3i}{2}e^{-2U} \int \Omega \wedge \bar{\Omega}$ one infers

$$e^U = \sqrt{2} e^{\frac{1}{2}(K^K - K^{cs})} , \quad (4.18)$$

where Kähler potential $K^K(t)$ is given in (3.34) while $K^{cs}(q)$ is the restriction of the Kähler potential (2.12) to the real slice $\mathcal{M}_{\mathbb{R}}^{cs}$. The existence of ϵ imposes constraints on the map φ . These BPS conditions read

$$\varphi^*(\Omega) = e^{U+i\theta_B} \sqrt{\det(\varphi^*(\hat{g} + \hat{B}_2) + 2\pi\alpha' F_2)} d^3\lambda , \quad \varphi^* J_c + i2\pi\alpha' F_2 = 0 , \quad (4.19)$$

where J_c is given in (3.11). The second condition in (4.19) enforces $\varphi^*(J) = 0$ as well as $\varphi^*\hat{B}_2 + 2\pi\alpha' F_2 = 0$, such that the first equation simplifies to

$$\varphi^* \text{Re}(e^{-i\theta_B} \Omega) = e^U \sqrt{\det(\varphi^*\hat{g})} d^3\lambda , \quad \varphi^* \text{Im}(e^{-i\theta_B} \Omega) = 0 , \quad (4.20)$$

where we have used that the volume element on \mathcal{W}_3 is real. The equations (4.19) and (4.20) imply that the Euclidean $D2$ branes have to wrap special Lagrangian cycles in Y , which are calibrated with respect to $\text{Re}(e^{-U-i\theta_B} \Omega)$. On the other hand, recall that the orientifold planes are located at the fixed points of the anti-holomorphic involution σ in Y which are special Lagrangian cycles calibrated with respect to $\text{Re}(e^{-U-i\theta} \Omega)$ as was argued in eqs. (3.4) and (3.5).²² Thus, in order for the D -instantons to preserve the same linear combination of the supercharges as the orientifold, we have to demand $\theta_B = \theta$. Using this constraint and inserting the calibration conditions (4.20) back into (4.17) one finds

$$S_{D2} = -2\mu_3 \int_{\mathcal{W}_3} \varphi^* [\text{Re}(C\Omega)] + i\mu_3 \int_{\mathcal{W}_3} \varphi^*(\hat{C}_3) = -i \int_{\mathcal{W}_3} \varphi^* \Omega_c , \quad (4.21)$$

where $C = \frac{1}{2}e^{-\phi-i\theta}e^{-U}$ was defined in eqs. (3.36), (2.7) and Ω_c is given in (3.49). The coefficients of Ω_c expanded in a basis of $H_+^3(Y)$ are exactly the $N = 1$ Kähler coordinates (N^k, T_λ) introduced in (3.47). As a consequence the instanton action (4.21) is linear and thus holomorphic in these coordinates which shows that $D2$ -instantons can correct the superpotential. Explicitly such corrections can be obtained by evaluating appropriate fermionic 2-point functions which are weighted by $e^{-S_{D2}}$ [25]. Applying (4.21) and keeping only the lowest term in the fluctuations of the instanton one obtains corrections of the form

$$W_{D3} \propto e^{i \int_{\Sigma_3} \Omega_c} , \quad (4.22)$$

where Σ_3 is the three-cycle wrapped by the $D2$ instanton. This result can be lifted to M-theory by embedding Calabi-Yau orientifolds into compactifications on special G_2 manifolds. In this case the $D2$ instantons correspond to membranes wrapping three-cycles in the G_2 space which do not extend in the dilaton direction [25, 26]. The embedding of IIA orientifolds into G_2 manifolds and the comparison of the respective effective actions is the subject of the next section.

²² e^{-U} is the normalization factor which was left undetermined in (3.5).

5 The G_2 embedding of IIA orientifolds

In this section we discuss the relationship between the type IIA Calabi-Yau orientifolds considered so far and G_2 compactifications of M-theory. In refs. [26] it was argued that for a specific class of G_2 compactifications X , type IIA orientifolds appear at special loci in their moduli space. More precisely, these G_2 manifolds have to be such that they admit the form

$$X = (Y \times S^1)/\hat{\sigma}, \quad (5.1)$$

where Y is a Calabi-Yau threefold and $\hat{\sigma} = (\sigma, -1)$ is an involution which inverts the coordinates of the circle S^1 and acts as an anti-holomorphic isometric involution on Y . σ and $\hat{\sigma}$ can have a non-trivial fixpoint set and as a consequence X is a singular G_2 manifold. In terms of the type IIA orientifolds the fixpoints of σ are the locations of the $O6$ planes in Y and as we already discussed earlier cancellation of the appearing tadpoles require the presence of appropriate $D6$ -branes. In this paper we froze all excitation of the $D6$ -branes and only discussed the effective action of the orientifold bulk. In terms of G_2 compactification this corresponds to the limit where X is smoothed out and all additional moduli arising in this process are frozen.

The purpose of this section is to check the embedding of type IIA orientifolds into G_2 compactifications of M-theory at the level of the $N = 1$ effective action. For orientifolds the effective action was derived in sections 3 and 4 and so as a first step we need to recall the effective action of M-theory (or rather eleven-dimensional supergravity) on smooth G_2 manifolds [27, 25, 30, 31, 32].

The only multiplet in eleven-dimensional supergravity is the supergravity multiplet, which consists of the metric g_{11} and a three-form C_3 as bosonic components. The effective action for these fields is given by [55]

$$S^{(11)} = \frac{1}{\kappa_{11}^2} \int \frac{1}{2} R * \mathbf{1} - \frac{1}{4} G_4 \wedge * G_4 - \frac{1}{12} C_3 \wedge G_4 \wedge G_4, \quad (5.2)$$

where $G_4 = dC_3$ is the field strength of C_3 . As in the reduction on Calabi-Yau manifolds one chooses the background metric to admit a block-diagonal form

$$ds^2 = ds_4^2(x) + ds_{G_2}^2(y), \quad (5.3)$$

where ds_4^2 and $ds_{G_2}^2$ are the line elements of a Minkowski and a G_2 metric, respectively. The Kaluza-Klein Ansatz for the three-form C_3 reads

$$C_3 = c^i(x) \phi_i + A^\alpha(x) \wedge \omega_\alpha, \quad i = 1, \dots, b^3(X), \quad \alpha = 1, \dots, b^2(X) \quad (5.4)$$

where c^i are real scalars and A^α are one-forms in four space-time dimensions. The harmonic forms ϕ_i and ω_α span a basis of $H^3(X)$ and $H^2(X)$, respectively. The G_2 holonomy allows for exactly one covariantly constant spinor which can be used to define a real, harmonic and covariantly constant three-form Φ .²³ The deformation space of the G_2 metric has dimension $b^3(X) = \dim H^3(X, \mathbb{R})$ and can be parameterized by expanding Φ into the basis ϕ_i [56]

$$\Phi = s^i(x) \phi_i. \quad (5.5)$$

²³The covariantly constant three-form is the analog of the holomorphic three-form Ω on Calabi-Yau manifolds.

One combines the real scalars s^i and c^i into complex coordinates according to

$$S^i = c^i + i s^i , \quad (5.6)$$

which form the bosonic components of $b^3(X)$ chiral multiplets. In addition the effective four-dimensional supergravity features $b^2(X)$ vector multiplets with the A^α as bosonic components. Due to the $N = 1$ supersymmetry, the couplings of these multiplet are again expressed in terms of a Kähler potential K_{G_2} , gauge-kinetic coupling functions f_{G_2} and a (flux induced) superpotential W_{G_2} . Let us discuss these functions in turn.

The Kähler potential was found to be [25, 30, 31, 32]

$$K_{G_2} = -3 \ln \left(\frac{1}{\kappa_{11}^2} \frac{1}{7} \int_X \Phi \wedge * \Phi \right) , \quad (5.7)$$

where $\frac{1}{7} \int \Phi \wedge * \Phi = \text{vol}(X)$ is the volume of the G_2 manifold X . The associated Kähler metric is given by

$$\partial_i \bar{\partial}_j K_{G_2} = \frac{1}{4} \text{vol}(X)^{-1} \int_X \phi_i \wedge * \phi_j , \quad \partial_i K_{G_2} = \frac{i}{2} \text{vol}(X)^{-1} \int_X \phi_i \wedge * \Phi , \quad (5.8)$$

and obeys the no-scale type condition

$$(\partial_i K_{G_2}) K_{G_2}^{i\bar{j}} (\partial_{\bar{j}} K_{G_2}) = 7 . \quad (5.9)$$

The holomorphic gauge coupling functions f_{G_2} arise from the couplings of C_3 in (5.2). At the tree level they are linear in S^i and read [25, 31]

$$(f_{G_2})_{\alpha\beta} = \frac{i}{2\kappa_{11}^2} S^i \int_X \phi_i \wedge \omega_\alpha \wedge \omega_\beta . \quad (5.10)$$

Finally, non-vanishing background flux of G_4 induces a scalar potential which via (3.31) can be expressed in terms of the superpotential [28, 29, 32]

$$W_{G_2} = \frac{1}{4\kappa_{11}^2} \int_X \left(\frac{1}{2} C_3 + i \Phi \right) \wedge G_4 . \quad (5.11)$$

(The factor 1/2 ensures holomorphicity of W_{G_2} in the coordinates S^i and compensates the quadratic dependence on C_3 [32].)

In order to compare the low energy effective theory of G_2 compactifications with the one of the orientifold we first have to restrict to the special G_2 manifolds X introduced in (5.1). This can be done by analyzing how the cohomologies of X are related to the ones of Y . As in equation (3.7) we consider the splits $H^p(Y) = H_+^p \oplus H_-^p$ of the cohomologies into eigenspaces of the involution σ . Working on the G_2 manifold X given in (5.1) we thus find the $\hat{\sigma}$ -invariant cohomologies

$$\begin{aligned} H^2(X) &= H_+^2(Y) , & H^3(X) &= H_+^3(Y) \oplus [H_-^2(Y) \wedge H_-^1(S^1)] , \\ H^5(X) &= H_-^4(Y) \wedge H_-^1(S^1) , & H^4(X) &= H_+^4(Y) \oplus [H_-^3(Y) \wedge H_-^1(S^1)] , \end{aligned} \quad (5.12)$$

where $H^2(X)$ and $H^5(X)$ as well as $H^3(X)$ and $H^4(X)$ are Hodge duals. $H_-^1(S^1)$ is the one-dimensional space containing the odd one-form of S^1 . The split of $H^3(X)$ induces a

split of the G_2 -form Φ which is most easily seen by introducing locally an orthonormal basis $(e^1, \dots, e^7) \in \Lambda^1(X)$ of one-forms. In terms of this basis one has [56, 30, 57]

$$\Phi = J_M \wedge e^7 + \text{Re}\Omega_M, \quad * \Phi = \frac{1}{2} J_M \wedge J_M + \text{Im}\Omega_M \wedge e^7, \quad (5.13)$$

where

$$J_M = e^1 \wedge e^2 + e^3 \wedge e^4 + e^5 \wedge e^6, \quad \Omega_M = (e^1 + ie^2) \wedge (e^3 + ie^4) \wedge (e^5 + ie^6). \quad (5.14)$$

Applied to the manifold (5.1) we may interpret $e^7 = dy^7$ as being the odd one-form along S^1 . Since Φ is required to be invariant under $\hat{\sigma}$ and σ is anti-holomorphic the decomposition (5.13) implies

$$\hat{\sigma}^* J_M = -J_M, \quad \hat{\sigma}^* \Omega_M = \bar{\Omega}_M. \quad (5.15)$$

In terms of the basis vectors e^1, \dots, e^6 this is ensured by choosing e^4, e^5, e^6 to be odd and e^1, e^2, e^3 to be even under σ . We see that J_M and Ω_M satisfy the exact same conditions as the corresponding forms of the orientifold (c.f. (3.2), (3.3)) and thus have to be proportional to J and $C\Omega$ used in section 3. In order to determine the exact relation it is necessary to fix their relative normalization. The relation between J_M and the Kähler form J in the string frame can be determined from the relation of the respective metrics. Reducing eleven-dimensional supergravity to type IIA supergravity in the string frame requires the line element (5.3) of the eleven-dimensional metric to take the form

$$ds^2 = e^{-2\hat{\phi}/3} ds_4^2(x) + e^{-2\hat{\phi}/3} g_{(s)ab} dy^a dy^b + e^{4\hat{\phi}/3} (dy^7)^2, \quad (5.16)$$

where $a, b = 1, \dots, 6$. The factors $e^{\hat{\phi}}$ of the ten-dimensional dilaton are chosen such that the type IIA supergravity action takes the standard form with $g_{(s)}$ being the Calabi-Yau metric in string frame (see e.g. [4]). Consequently we have to identify

$$J_M = e^{-2\hat{\phi}/3} J. \quad (5.17)$$

Similarly, using (5.14) we find that the normalization of Ω_M is given by

$$J_M \wedge J_M \wedge J_M = \frac{3i}{4} \Omega_M \wedge \bar{\Omega}_M. \quad (5.18)$$

Integrating over Y and using (5.17), (3.34) and (2.12) we obtain

$$\Omega_M = e^{-\hat{\phi}-i\theta} e^{\frac{1}{2}(K^{\text{cs}}-K^{\text{K}})} \Omega = \sqrt{8} C \Omega, \quad (5.19)$$

where C is given in (3.36). The phase $e^{i\theta}$ drops out in (5.18) such that we can choose it as in (3.3) in order to fulfill (5.15). Inserting J_M and Ω_M into equation (5.15) one arrives at

$$\Phi = J \wedge d\tilde{y}^7 + \sqrt{8} \text{Re}(C\Omega), \quad (5.20)$$

where we defined $d\tilde{y}^7 = e^{-\frac{2\hat{\phi}}{3}} dy^7$. The form $d\tilde{y}^7$ is normalized such that $\int_{S^1} d\tilde{y}^7 = 2\pi R$ where the metric (5.16) was used and R is the ϕ -independent radius of the internal circle. We also set $\kappa_{10}^2 = \kappa_{11}^2 / 2\pi R = 1$ henceforth. Using (5.20), (5.13) and (3.36) we calculate

$$\frac{1}{\kappa_{11}^2} \frac{1}{7} \int \Phi \wedge * \Phi = e^{-\frac{4\hat{\phi}}{3}} \frac{1}{6} \int J \wedge J \wedge J, \quad (5.21)$$

which equivalently can be obtained by applying the volume split $\text{vol}(X) = \text{vol}(Y) \cdot \text{vol}(S^1)$ evaluated in the metric (5.16). Inserting (5.21) into (5.7) using (3.36) we obtain

$$K_{G_2} = -\ln \left[\frac{1}{6} \int J \wedge J \wedge J \right] - 2 \ln \left[2 \int_Y \text{Re}(C\Omega) \wedge *_6 \text{Re}(C\Omega) \right]. \quad (5.22)$$

Thus we find exactly the Kähler potential K of the type IIA orientifold as given in (3.48).²⁴

In order to compare the gauge kinetic functions and the superpotential we also need to identify the Kähler coordinates of the two theories. C_3 splits under the decomposition (5.12) of the cohomologies as²⁵

$$C_3 = \hat{B}_2 \wedge d\tilde{y}^7 + \sqrt{2} \hat{C}_3, \quad (5.23)$$

where \hat{B}_2 is an odd two-form on Y and \hat{C}_3 an even three-form on Y . Combining (5.20) and (5.23) using (3.49) one finds

$$S^i \phi_i = C_3 + i\Phi = J_c \wedge d\tilde{y}^7 + \sqrt{2} \Omega_c. \quad (5.24)$$

As discussed after (3.49) the coefficients arising in the expansions of J_c and Ω_c into the basis $(\alpha_k, \beta^\lambda)$ of $H_+^3(Y)$ and ω_a of $H^2(Y)$ are exactly the orientifold coordinates and therefore we have to identify $S^a \cong t^a$ and $S^K \cong (N^k, T_\lambda)$. With this information at hand, it is not difficult to show that the gauge-kinetic couplings (5.10) coincide with (3.32). One splits $\phi_a = \omega_a \wedge d\tilde{y}^7$ and obtains

$$(f_{G_2})_{\alpha\beta} = \frac{i}{2} S^a \int_Y \omega_a \wedge \omega_\alpha \wedge \omega_\beta \sim i t^a \mathcal{K}_{a\alpha\beta} = (f_{OY})_{\alpha\beta}, \quad (5.25)$$

where the precise factor depends on the normalization of the gauge fields.

It remains to compare the flux induced superpotentials (5.11) with (4.15). Using the cohomology splits (5.12) and (5.23) the background flux splits accordingly as $G_4 = H_3 \wedge d\tilde{y}^7 + \sqrt{2} F_4$. Inserted into (5.11) using (5.24) we arrive at

$$W_{G_2} = \frac{1}{\sqrt{8}} \int_Y J_c \wedge F_4 + \frac{1}{\sqrt{8}} \int_Y \Omega_c \wedge H_3 \quad (5.26)$$

Compared to (4.15) the superpotential W_{G_2} only includes terms proportional to the fluxes H_3 and F_4 .²⁶ The remaining terms in (4.15) should arise once manifolds with G_2 structure (instead of G_2 holonomy) are considered. However, the discussion of this generalization is beyond the scope of this paper.

6 Mirror symmetry

In this section we discuss mirror symmetry for Calabi-Yau orientifolds from the point of view of the effective action derived in the large volume limit. More precisely, we compare

²⁴In terms of the Hitchin functionals [30] recently discussed in [58, 59] the reduction of the G_2 Kähler potential (5.7) corresponds to the split of the seven-dimensional Hitchin functional to the two six-dimensional ones 5.22.

²⁵We have introduced a factor of $\sqrt{2}$ for later convenience.

²⁶The term proportional to e_0 in (4.16) can be absorbed into a redefinition of Ret^a [32].

the $N = 1$ data obtained in the previous sections for type IIA orientifolds with the ones determined in ref. [21] for type IIB orientifolds. In order to do so we need to briefly review some properties of type IIB Calabi-Yau orientifolds [8, 10, 21].

Similar to type IIA the type IIB orientifolds are obtained by modding out IIB string theory compactified on a Calabi-Yau manifold \tilde{Y} by a discrete symmetry \mathcal{O} which is involutive $\mathcal{O}^2 = 1$ and includes worldsheet parity Ω_p . For type IIB one has two distinct choices for \mathcal{O} depending on the transformation properties of the Calabi-Yau three-form Ω . They are given by [8, 10]

$$\begin{aligned} \mathcal{O}_1 &= \Omega_p \sigma_B (-)^{F_L} , & \sigma_B^* \Omega &= -\Omega , & O3/O7 , \\ \mathcal{O}_2 &= \Omega_p \sigma_B , & \sigma_B^* \Omega &= \Omega , & O5/O9 . \end{aligned} \quad (6.1)$$

Modding out by \mathcal{O}_1 leads to the presence of $O3/O7$ planes while modding out by \mathcal{O}_2 results in $O5/O9$ planes. σ_B is again an involutive symmetry $\sigma_B^2 = 1$ which acts on the Calabi-Yau coordinates but in contrast to the situation in type IIA it is a holomorphic isometry of \tilde{Y} and therefore obeys in both cases $\sigma_B^* J = J$.

The $N = 1$ spectrum is obtained from the invariant modes of the ten-dimensional type IIB fields $\phi_B, \hat{C}_0, \hat{B}_2, \hat{C}_2$ and \hat{C}_4 . Without repeating the details one finds that in analogy to (3.6) the invariant modes have to transform according to [10]

$$\begin{array}{ccc} & \underline{O3/O7} & \underline{O5/O9} \\ \sigma_B^* \hat{\phi} & = & \hat{\phi} , \\ \sigma_B^* \hat{g} & = & \hat{g} , \\ \sigma_B^* \hat{B}_2 & = & -\hat{B}_2 , \\ & \sigma_B^* \hat{C}_0 & = \hat{C}_0 , \\ & \sigma_B^* \hat{C}_2 & = -\hat{C}_2 , \\ & \sigma_B^* \hat{C}_4 & = \hat{C}_4 , \end{array} \quad (6.2)$$

where the first column is identical for both involutions σ_B in (6.1).

Since σ_B is a holomorphic involution the cohomologies of \tilde{Y} split again into eigenspaces of σ_B as

$$H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)} . \quad (6.3)$$

In the Kaluza-Klein reduction on \tilde{Y} , the ten-dimensional fields are expanded in harmonic forms in the appropriate eigenspaces of σ_B . Inserting these expansions into the ten-dimensional IIB supergravity action results in an $N = 1$ supergravity in $d = 4$ which can be brought into the form (3.30) and therefore is characterized by a Kähler potential, a set of gauge-kinetic functions and a superpotential. For both cases ($O3/O7$ and $O5/O9$) these $N = 1$ data have been determined in ref. [21] and we recall the results as we go along.

Analogously to (3.33) the moduli space of type IIB orientifolds locally is a direct product of two Kähler manifolds

$$\tilde{\mathcal{M}}_B^K \times \tilde{\mathcal{M}}_B^Q , \quad (6.4)$$

where $\tilde{\mathcal{M}}_B^K$ is again a special Kähler manifold obtained by reducing the type IIB $N = 2$ special Kähler manifold while $\tilde{\mathcal{M}}_B^Q$ is a Kähler subspace of the $N = 2$ quaternionic manifold. However, in type IIB the manifold $\tilde{\mathcal{M}}_B^K$ is spanned by the complex structure deformations of \tilde{Y} respecting the constraints (6.1). This implies that it can be parameterized by $h_-^{(2,1)}$ complex scalars z^a for orientifolds with $O3/O7$ planes and $h_+^{(2,1)}$ complex

scalars z^α for orientifolds with $O5/O9$ planes. $\tilde{\mathcal{M}}_B^Q$ has complex dimension $h^{(1,1)} + 1$ for both type IIB theories and includes the type IIB dilaton, the Kähler deformation of \tilde{Y} and the scalars arising from \hat{B}_2, \hat{C}_2 and \hat{C}_4 . Additionally the IIB effective theory contains $h_+^{(2,1)}(h_-^{(2,1)})$ vector multiplets for orientifolds with $O3/O7(O5/O9)$ planes. We summarize the number of chiral and vector multiplets in table 6.1.

multiplets	IIA _Y $O6$	IIB _{\tilde{Y}} $O3/O7$	IIB _{\tilde{Y}} $O5/O9$
vector multiplets	$h_+^{(1,1)}$	$h_+^{(2,1)}$	$h_-^{(2,1)}$
chiral multiplets in $\tilde{\mathcal{M}}^K$	$h_-^{(1,1)}$	$h_-^{(2,1)}$	$h_+^{(2,1)}$
chiral multiplets in $\tilde{\mathcal{M}}^Q$	$h^{(2,1)} + 1$	$h^{(1,1)} + 1$	$h^{(1,1)} + 1$

Table 6.1: Number of $N = 1$ multiplets of orientifold compactifications.

Since we want to discuss mirror symmetry we choose \tilde{Y} to be the mirror manifold of Y . This implies that the non-trivial Hodge numbers $h^{(1,1)}$ and $h^{(2,1)}$ of Y and \tilde{Y} satisfy

$$h^{(1,1)}(Y) = h^{(2,1)}(\tilde{Y}), \quad h^{(2,1)}(Y) = h^{(1,1)}(\tilde{Y}). \quad (6.5)$$

In addition, we also have to specify the involutions σ_A and σ_B which are identified under mirror symmetry. Since the discussion in this paper is quite generic and never specified any involution σ explicitly we also keep the discussion of mirror symmetry generic. That is we assume that there exists a mirror pair of manifolds Y and \tilde{Y} with a mirror pair of involutions σ_A, σ_B . This implies an orientifold version of (6.5),²⁷ i.e.

$$\begin{aligned} O3/O7 : \quad h_-^{1,1}(Y) &= h_-^{2,1}(\tilde{Y}), & h_+^{1,1}(Y) &= h_+^{2,1}(\tilde{Y}), \\ O5/O9 : \quad h_-^{1,1}(Y) &= h_+^{2,1}(\tilde{Y}), & h_+^{1,1}(Y) &= h_-^{2,1}(\tilde{Y}). \end{aligned} \quad (6.6)$$

Our next task will be to match the couplings of the mirror theories. Since the effective actions on both sides are only computed in the large volume limit we can expect to find agreement only if we also take the large complex structure limit exactly as in the $N = 2$ mirror symmetry. However, if one believes in mirror symmetry one can use the geometrical results of the complex structure moduli space to ‘predict’ the corrections to its mirror symmetric component. This is not quite as straightforward since the full $N = 1$ moduli space is a lot more complicated than the underlying $N = 2$ space [10]. Let us therefore start our analysis with the simpler situation of the special Kähler sectors $\tilde{\mathcal{M}}_A^K, \tilde{\mathcal{M}}_B^K$ and the vector multiplet couplings and postpone the analysis of $\tilde{\mathcal{M}}_{A,B}^Q$ to sections 6.2.1 and 6.2.2.

6.1 Mirror symmetry in $\tilde{\mathcal{M}}^K$

Recall that the manifold $\tilde{\mathcal{M}}_A^K$ is spanned by the complexified Kähler deformations t^a preserving the constraint (3.2). Under mirror symmetry these moduli are mapped to the

²⁷For the sector of $\tilde{\mathcal{M}}^Q$ mirror symmetry is a constraint on the couplings rather than the Hodge numbers.

complex structure deformations which respect the constraint (6.1). In both cases the Kähler potential is merely a truncated version of the $N = 2$ Kähler potential and one has

$$K_A^K = -\ln \left[\frac{4}{3} \int_Y J \wedge J \wedge J \right] \leftrightarrow K_B^{\text{cs}} = -\ln \left[-i \int \Omega \wedge \bar{\Omega} \right]. \quad (6.7)$$

Both Kähler potentials can be expressed in terms of prepotentials $f_A(t), f_B(z)$ and in the large complex structure limit $f_B(z)$ becomes cubic and agrees with $f_A(t)$. Mirror symmetry therefore equates these prepotentials and exchanges J^3 with $\Omega \wedge \bar{\Omega}$ exactly as in $N = 2$

$$f_A(t) = f_B(z), \quad J^3 \leftrightarrow \Omega \wedge \bar{\Omega}. \quad (6.8)$$

Thus for $\tilde{\mathcal{M}}^K$ mirror symmetry is a truncated version of $N = 2$ mirror symmetry. As we will see momentarily this also holds for the couplings (the gauge kinetic couplings and the superpotential) which depend on the moduli spanning $\tilde{\mathcal{M}}^K$.

In type IIA the gauge-kinetic couplings are given in (3.32) and read $f_{\alpha\beta}(t) = i\mathcal{K}_{\alpha\beta}t^c$. The IIB couplings were determined in ref. [21] to be

$$f_{\alpha\beta}(z^a) = -i\tilde{\mathcal{M}}_{\alpha\beta} = -i\mathcal{F}_{\alpha\beta}, \quad (6.9)$$

where in order to not overload the notation we are using the same indices for both cases.²⁸ More precisely we are choosing

$$\begin{aligned} \alpha, \beta = 1, \dots, h_+^{(2,1)}(\tilde{Y}), \quad a, b = 1, \dots, h_-^{(2,1)}(\tilde{Y}), & \quad \text{for } O3/O7, \\ \alpha, \beta = 1, \dots, h_-^{(2,1)}(\tilde{Y}), \quad a, b = 1, \dots, h_+^{(2,1)}(\tilde{Y}), & \quad \text{for } O5/O9. \end{aligned} \quad (6.10)$$

The matrix $\mathcal{F}_{\alpha\beta}(z^a)$ is holomorphic and the second derivatives of the prepotential restricted to $\tilde{\mathcal{M}}_B^K$. In the large complex structure limit $\mathcal{F}_{\alpha\beta}$ is linear in z^a and therefore also agrees with the type IIA mirror couplings. Thus mirror symmetry implies the map

$$\mathcal{N}_{\alpha\beta}(\bar{t}^a) = \mathcal{M}_{\alpha\beta}(\bar{z}^a), \quad (6.11)$$

in both cases.

It is also straightforward to match the superpotentials which are induced by RR background flux. For both type IIB cases they are given by [12]

$$W_B(z^a) = \int_{\tilde{Y}} \Omega \wedge F_3, \quad (6.12)$$

where F_3 is the flux of the field strength of \hat{C}_2 . The two-form \hat{C}_2 transforms differently in the two IIB orientifolds as can be seen in (6.2). Therefore F_3 sits in $H_-^3(\tilde{Y})$ and is determined in terms of $2h_-^{(2,1)} + 2$ real flux parameters for the $O3/O7$ case and sits in $H_+^3(\tilde{Y})$ depending on $2h_+^{(2,1)} + 2$ real flux parameters for the $O5/O9$ case. On the IIA side the superpotential $W^K(t^a)$ is given in (4.16) and can be succinctly written as [12, 28]

$$W_A(t) = \int e^{J_c} \wedge F_{RR}, \quad (6.13)$$

²⁸We rescaled the type IIB gauge bosons by $\sqrt{2}$ in order to properly match the normalizations.

where F_{RR} stands for a formal sum over all even RR-fluxes. It depends on $2h_-^{(1,1)} + 2$ RR fluxes $(e_{\hat{a}}, m^{\hat{a}})$ in agreement with (6.6). Furthermore, the functional dependence of the superpotentials coincide under the mirror map (6.8) which more generally can also be written as [60]

$$e^{J_c}(t) \leftrightarrow \Omega(z) , \quad F_{RR} \leftrightarrow F_3 . \quad (6.14)$$

This concludes our discussions of mirror symmetry for the chiral multiplets which span $\tilde{\mathcal{M}}^K$. We have shown that the Kähler potential, the gauge-kinetic coupling functions and the RR superpotential agree in the large complex structure limit under mirror symmetry. In this sector the geometrical quantities on the type IIB side include corrections which are believed to compute worldsheet non-perturbative effects such as worldsheet instantons on the type IIA side. This is analogous to the situation in $N = 2$ and may be traced back to the fact, that it is still possible to formulate a topological A model counting world-sheet instantons for Calabi-Yau orientifolds [8, 49].

6.2 Mirror symmetry in $\tilde{\mathcal{M}}^Q$

Let us now turn to the discussion of the Kähler manifolds $\tilde{\mathcal{M}}_A^Q$ and $\tilde{\mathcal{M}}_B^Q$ arising in the reduction of the quaternionic spaces. On the IIA side the Kähler potential is given in (3.48) which is expressed in terms of the $h^{(2,1)} + 1$ coordinates (N^k, T_λ) defined in (3.47). In this definition we did not fix the scale invariance (3.14) $\Omega \rightarrow \Omega e^{-\text{Re}(h)}$ or in other words we defined the coordinates in terms of the scale invariant combination $C\Omega$. Somewhat surprisingly there seem to be two physically inequivalent ways to fix this scale invariance. In $N = 2$ one uses the scale invariance to define special coordinates $z^K = Z^K/Z^0, z^0 = 1$ where Z^0 is the coefficient in front of the base element α_0 . The choice of Z^0 is convention and due to the symplectic invariance any other choice would be equally good. However, as we already discussed in section 3.1 and 3.3 the constraint (3.3) breaks the symplectic invariance and H^3 decomposes into two eigenspaces $H_+^3 \oplus H_-^3$. Thus in (3.46) we have the choice to scale one of the Z^k equal to one or one of the Z^λ equal to i . Denoting the corresponding basis element by α_0 , these two choices are characterized by $\alpha_0 \in H_+^3$ or $\alpha_0 \in H_-^3$. This choice identifies the dilaton direction inside the moduli space and therefore is crucial in identifying the type IIB mirror. This is related to the fact that in type IIB the dilaton reside in a chiral multiplet for $O3/O7$ orientifolds and in a linear multiplet for $O5/O9$ orientifolds. Let us discuss these two cases in turn.

6.2.1 The Mirror of IIB orientifolds with $O3/O7$ planes

For type IIB Calabi-Yau orientifolds with $O3/O7$ planes the low energy theory was derived in ref. [21]. The Kähler manifold $\tilde{\mathcal{M}}_B^Q$ is spanned by $h^{(1,1)}(\tilde{Y}) + 1$ chiral multiplets which arise from the expansion of J, \hat{B}_2, \hat{C}_2 and \hat{C}_4

$$\begin{aligned} \hat{B}_2 &= b^k(x) \omega_k , & \hat{C}_2 &= c^k(x) \omega_k , & k &= 1, \dots, h_-^{(1,1)}(\tilde{Y}) , \\ J &= v^\lambda(x) \omega_\lambda , & \hat{C}_4 &= \rho_\lambda(x) \tilde{\omega}^\lambda , & \lambda &= 1, \dots, h_+^{(1,1)}(\tilde{Y}) , \end{aligned} \quad (6.15)$$

where we only displayed the scalar fields in the expansion. The proper Kähler coordinates were identified as²⁹

$$\begin{aligned}\tau &= C_0 + ie^{-\phi_B}, & G^k &= c^k - \tau b^k, \\ T_\lambda &= 2i\rho_\lambda + e^{-\phi_B} \mathcal{K}_{\lambda\rho\sigma} v^\rho v^\sigma - i\mathcal{K}_{\lambda kl} b^k G^l,\end{aligned}\quad (6.16)$$

where C_0 is the RR scalar and e^{ϕ_B} is the type IIB dilaton. The intersection numbers $\mathcal{K}_{\lambda\rho\sigma}$ and $\mathcal{K}_{\lambda kl}$ are defined exactly as in (2.18) and are the only non-vanishing intersections of the even cohomologies in IIB orientifolds. The Kähler potential is given by

$$K_B^Q(\tau, G, T) = -2 \ln \left[e^{-2\phi_B} \int J \wedge J \wedge J \right] = -\ln(e^{-4D_B}), \quad (6.17)$$

where e^{D_B} is the four-dimensional dilaton. K^Q can only be given implicitly as a function of v^λ and e^{ϕ_B} which are determined by (6.16) in terms of the variables τ, T_λ and G^k .

Now we want to show that in the large complex structure limit K_A^Q given in (3.42) coincides with K_B^Q given in (6.17). It turns out that in order to do so we need to choose $\alpha_0 \in H_+^3$ and the dual basis element $\beta^0 \in H_-^3$. It is convenient to keep track of this choice and therefore we mark the α 's and β 's which contain α_0 and β^0 by putting a hat on the corresponding index. Thus we work in the basis $(\alpha_{\hat{k}}, \beta^\lambda)$ of H_+^3 and $(\alpha_\lambda, \beta^{\hat{k}})$ of H_-^3 . Therefore, we rewrite the combination $C\Omega$ as

$$C\Omega = g_A^{-1}(\mathbf{1} \alpha_0 + q^k \alpha_k + iq^\lambda \alpha_\lambda) + \dots, \quad (6.18)$$

where we introduced g_A and the real special coordinates

$$g_A = \frac{1}{\text{Re}(CZ^0)}, \quad q^k = \frac{\text{Re}(CZ^k)}{\text{Re}(CZ^0)}, \quad q^\lambda = \frac{\text{Im}(CZ^\lambda)}{\text{Re}(CZ^0)}. \quad (6.19)$$

We also need to express the prepotential $\mathcal{F}(Z)$ in the special coordinates q^k, q^λ . In analogy to (A.9) one defines a function $f(q)$ such that

$$\mathcal{F}(\text{Re}[CZ^{\hat{k}}], i\text{Im}[CZ^\lambda]) = i(\text{Re}[CZ^0])^2 f(q^k, q^\lambda). \quad (6.20)$$

We are now in the position to rewrite the $N = 1$ coordinates $N^{\hat{k}}, T_\lambda$ given in (3.47) in terms of g_A and the special coordinates q^K . Inserting (6.19) into (3.47) one obtains

$$N^0 = \frac{1}{2}\xi^0 + ig_A^{-1}, \quad N^k = \frac{1}{2}\xi^k + ig_A^{-1}q^k, \quad T_\lambda = i\tilde{\xi}_\lambda - 2g_A^{-1}f_\lambda(q), \quad (6.21)$$

where f_λ is the first derivative of $f(q)$ with respect to q^λ .

The final step is to specify $f(q)$ in the large complex structure limit. In this limit the $N = 2$ prepotential is known to be

$$\mathcal{F}(Z) = \frac{1}{6}(Z^0)^{-1} \kappa_{KLM} Z^K Z^L Z^M. \quad (6.22)$$

Inserted into the orientifold constraints (3.45) one infers

$$\kappa_{klm} = \kappa_{\kappa\lambda l} = 0, \quad (6.23)$$

²⁹We have slightly changed the conventions with respect to [21], since the scalars v^α are now given in string frame.

while $\kappa_{\kappa\lambda\mu}$ and κ_{klm} can be non-zero. Using (6.23), (6.20) and (6.19) we arrive at

$$f(q) = -\frac{1}{6}\kappa_{\kappa\lambda\mu}q^\kappa q^\lambda q^\rho + \frac{1}{2}\kappa_{\kappa kl}q^\kappa q^k q^l . \quad (6.24)$$

In order to continue we also have to specify the range the indices k and λ take on the IIA side. A priori it is not fixed and can be changed by a symplectic transformation. Mirror symmetry demands

$$k = 1, \dots, h_-^{(1,1)}(\tilde{Y}) , \quad \lambda = 1, \dots, h_+^{(1,1)}(\tilde{Y}) , \quad (6.25)$$

or in other words there have to be $h_-^{(1,1)}(\tilde{Y})$ basis elements α_k and $h_+^{(1,1)}(\tilde{Y})$ basis elements β^λ in $H_+^3(Y)$. In addition the non-vanishing couplings $\kappa_{\kappa\lambda\mu}$ and κ_{klm} have to be identified with $\mathcal{K}_{\kappa\lambda\mu}$ and \mathcal{K}_{klm} appearing in the definition of the type IIB chiral coordinates (6.16). With these conditions fulfilled we can insert (6.24) into (6.21) and compare with (6.16). This leads to the identification

$$\hat{N}^k = (\tau, G^k) \quad \text{and} \quad T_\lambda^A = T_\lambda^B , \quad (6.26)$$

which in terms of the Kaluza-Klein variables corresponds to

$$\begin{aligned} e^{\phi_B} &= g_A , & q^\lambda &= v^\lambda , & q^k &= -b^k , \\ \xi_0 &= 2C_0 , & \xi^k &= 2(c^k - C_0 b^k) , \\ \tilde{\xi}_\lambda &= 2\rho_\lambda - \mathcal{K}_{\lambda kl} c^k b^l + C_0 \mathcal{K}_{\lambda kl} b^k b^l . \end{aligned} \quad (6.27)$$

With these identifications one immediately shows $e^{D_A} = e^{D_B}$, where e^{D_A} and e^{D_B} are the four-dimensional dilatons of the type IIA and IIB theory. This implies that the Kähler potentials (3.42) and (6.17) of the two theories coincide in the large volume – large complex structure limit. However, the corrections away from this limit cannot be properly understood from a pure supergravity analysis. It is clear that K_A^Q includes corrections of the mirror IIB theory but the precise nature of these corrections remains to be understood.

6.2.2 The Mirror of IIB orientifolds with $O5/O9$ planes

In this section we check mirror symmetry for type IIB orientifolds with $O5/O9$ planes. As in the previous section we first need to briefly recall the results of ref. [21]. In this case the Kaluza-Klein expansion of the ten-dimensional type IIB fields change as a consequence of the different transformation properties given in (6.2) and (6.15) is replaced by

$$\begin{aligned} J &= v^k(x) \omega_k , & \hat{C}_2 &= C_2(x) + c^k(x) \omega_k , & k &= 1, \dots, h_+^{(1,1)}(\tilde{Y}) , \\ \hat{B}_2 &= b^\lambda(x) \omega_\lambda , & \hat{C}_4 &= \rho_\lambda(x) \tilde{\omega}^\lambda , & \lambda &= 1, \dots, h_-^{(1,1)}(\tilde{Y}) . \end{aligned} \quad (6.28)$$

The proper Kähler coordinates which span $\tilde{\mathcal{M}}^Q$ are the $h^{(1,1)} + 1$ chiral fields

$$\begin{aligned} t^k &= -ie^{-\phi_B} v^k + c^k , \\ A_\lambda &= 2i\mathcal{K}_{\lambda\rho k} b^\rho t^k + 2i\rho_\lambda , \\ S &= \frac{1}{3}e^{-\phi_B} \mathcal{K} + 2ih - \frac{1}{2}b^\lambda A_\lambda , \end{aligned} \quad (6.29)$$

where h is a scalar dual to the four-dimensional two-form C_2 defined in (6.28) and $\mathcal{K} = \mathcal{K}_{\lambda\kappa\rho}v^\lambda v^\kappa v^\rho$. The Kähler potential has the exact same form as for the $O3/O7$ case and is again given by (6.17) but this time it depends implicitly on the variables S, t^k, A_λ defined in (6.29).

In order to find the same chiral data on the IIA side, we have to examine the case where $\alpha_0 \in H_-^3$. Therefore we choose a basis $(\alpha_k, \beta^{\hat{\lambda}})$ of H_+^3 and $(\alpha_{\hat{\lambda}}, \beta^k)$ of H_-^3 . We rewrite the combination $C\Omega$ in this basis as

$$C\Omega = g_A^{-1}(i\alpha_0 + iq^\lambda\alpha_\lambda + q^k\alpha_k) + \dots \quad (6.30)$$

where we introduced the real special coordinates

$$g_A = \frac{1}{\text{Im}(CZ^0)}, \quad q^k = \frac{\text{Re}(CZ^k)}{\text{Im}(CZ^0)}, \quad q^\lambda = \frac{\text{Im}(CZ^\lambda)}{\text{Im}(CZ^0)}. \quad (6.31)$$

Let us also express the prepotential $\mathcal{F}(Z)$ in terms of q^k, q^λ . As in $N = 2$ one defines a function $f(q)$ such that

$$\mathcal{F}(\text{Re}[CZ^k], i\text{Im}[CZ^{\hat{\lambda}}]) = -i(\text{Im}[CZ^0])^2 f(q^k, q^\lambda). \quad (6.32)$$

We can now rewrite the $N = 1$ coordinates $T_{\hat{\lambda}}, N^k$ given in (3.47) in terms of q^k, q^λ and g_A as

$$\begin{aligned} N^k &= \frac{1}{2}\xi^k + ig_A^{-1}q^k, & T_\lambda &= i\tilde{\xi}_\lambda + 2g_A^{-1}f_\lambda(q), \\ T_0 &= i\tilde{\xi}_0 + 2g_A^{-1}(2f(q) - f_\lambda q^\lambda - f_k q^k), \end{aligned} \quad (6.33)$$

where f_λ, f_k are the first derivatives of $f(q)$ with respect to q^λ and q^k .

Going to the large complex structure limit, the $N = 2$ prepotential takes the form (6.22). We split the indices as $K = (k, \hat{\lambda})$ and apply the constraints (3.45) to find that

$$\kappa_{\kappa\lambda\mu} = \kappa_{\kappa kl} = 0 \quad \kappa_{klm} \neq 0, \quad \kappa_{\kappa\lambda l} \neq 0. \quad (6.34)$$

Using (6.34) and (6.32) we can calculate $f(q)$ as

$$f(q) = \frac{1}{6}\kappa_{klm}q^kq^lq^m - \frac{1}{2}\kappa_{\kappa\lambda k}q^\kappa q^\lambda q^k. \quad (6.35)$$

In order to match the chiral coordinates T_0, T_λ, N^k with the type IIB coordinates of (6.29) we need again to specify the range of the indices on the type IIA side. Obviously we need

$$k = 1, \dots, h_+^{(1,1)}(\tilde{Y}), \quad \lambda = 1, \dots, h_-^{(1,1)}(\tilde{Y}), \quad (6.36)$$

which is the equivalent of (6.25) with the plus and minus sign interchanged. Thus the non-vanishing intersections can be identified with \mathcal{K}_{klm} and $\mathcal{K}_{\kappa\lambda k}$ on the IIB side. Inserting $f(q)$ back into the equations (6.33) for the chiral coordinates $N^k, T_{\hat{\lambda}}$ and demanding (6.36) one can compare these to the type IIB coordinates (6.29). One identifies

$$T_{\hat{\lambda}} = (S, A_\lambda), \quad N^k = t^k. \quad (6.37)$$

In terms of the Kaluza-Klein modes this amounts to the identification

$$\begin{aligned} g_A &= e^{\phi_B}, & q^k &= -v^k, & q^\lambda &= b^\lambda, & \xi^k &= 2c^k, \\ \tilde{\xi}_\lambda &= 2\mathcal{K}_{\lambda\kappa l}c^l b^\kappa + 2\rho_\lambda, & \tilde{\xi}_0 &= 2h - \mathcal{K}_{l\lambda\kappa}c^l b^\lambda b^\kappa - \rho_\lambda b^\lambda. \end{aligned} \quad (6.38)$$

With these identifications one shows again $e^{D_A} = e^{D_B}$ and as a consequence the Kähler potentials agree in the large volume – large complex structure limit.

In summary, we found that it is indeed possible to obtain both type IIB setups as mirrors of the type IIA orientifolds discussed in section 3. In analogy to (6.14) we found in the $\tilde{\mathcal{M}}^Q$ component the mirror relation

$$\begin{aligned} O3/O7: \quad \text{Re}(C\Omega) &\leftrightarrow e^{-\phi_B} \text{Re } e^{J_c}, & C_3 &\leftrightarrow C_{\text{RR}} \wedge e^{-\hat{B}_2}, \\ O5/O9: \quad \text{Re}(C\Omega) &\leftrightarrow e^{-\phi_B} \text{Im } e^{J_c}, & C_3 &\leftrightarrow C_{\text{RR}} \wedge e^{-\hat{B}_2}. \end{aligned} \quad (6.39)$$

However, the crucial role of the two definitions of special coordinates remains to be understood further.

Using the correspondence 6.39 we can extend the observation of section 4 that the proper chiral coordinates ‘linearize’ the corresponding D-brane instanton action also to type IIB orientifolds [21]. One can define the form

$$A_p = (C_{\text{RR}} \wedge e^{-\hat{B}_2})_p + i e^{-\phi_B} \text{Cal}_p, \quad (6.40)$$

where the instantons are calibrated with respect to the p -form Cal_p . $(C_{\text{RR}} \wedge e^{-\hat{B}_2})_p$ is a p -form constructed out of the formal sum of the ten-dimensional RR forms present in the orientifold theory. Expanding A_p in terms of $H_+^{(p)}(Y)$ results in chiral coordinates which linearize the $D(p-1)$ instanton action. These coordinates can already be discovered in the orientifold theory since the D-branes are constructed such that they preserve the same $N = 1$ supersymmetry as the orientifolds.

7 Conclusions

In this paper we calculated the four-dimensional effective action of type IIA Calabi-Yau orientifolds in the presence of background fluxes. We restricted ourselves to Calabi-Yau spaces admitting an anti-holomorphic involutive symmetry which preserves $N = 1$ supersymmetry. The string theory is modded out by an involutive symmetry which includes this geometric symmetry and thus imposes constraints on the spectrum and the couplings of the theory.

We computed the effective action by a Kaluza-Klein analysis valid in the large volume limit and determined the chiral variables, the Kähler potential, the gauge kinetic function and the flux-induced superpotential at the tree level. We found that the moduli space of the $N = 1$ theory inherits a product structure $\tilde{\mathcal{M}}^K \times \tilde{\mathcal{M}}^Q$ from the underlying $N = 2$ theory obtained by ordinary Calabi-Yau compactification of type IIA. $\tilde{\mathcal{M}}^K$ is a special Kähler manifold parameterized by the complexified Kähler form J_c which decends from the $N = 2$ vector multiplets. The second component $\tilde{\mathcal{M}}^Q$ is parameterized by the periods of the ‘new’ three-form Ω_c ($= C_3 + 2i\text{Re}C\Omega$) containing the complex structure deformations of the Calabi-Yau orientifold. It is a Kähler submanifold inside the quaternionic manifold of $N = 2$ and has a geometric structure similar to the one of the moduli space of supersymmetric Lagrangian submanifolds [23].

A superpotential W is induced once background fluxes are turned on which depends on all geometrical moduli. It splits into the sum of two terms with one term depending

on the RR fluxes and the complexified Kähler form J_c while the second term features the NS fluxes and Ω_c . Both terms are expected to receive non-perturbative corrections from worldsheet- and D-brane instantons. We showed that the respective actions are linear in the chiral coordinates and therefore can result in holomorphic corrections to W .

We further discussed the embedding of type IIA orientifolds into a specific class of G_2 compactification of M-theory. Neglecting the contributions arising from the singularities of the G_2 manifold we were able to show agreement between the low energy effective actions. In the superpotential we only discovered the terms which descend from the M-theory four-form G_4 but we neglected the possibility of geometrical fluxes.

Finally we showed that in the large volume – large complex structure limit one finds mirror symmetric effective actions if one compares type IIA and type IIB supergravity compactified on mirror manifolds and in addition chooses a set of ‘mirror involutions’. For $\tilde{\mathcal{M}}^K$ mirror symmetry amounts to a truncated versions of $N = 2$ mirror symmetry in that it still relates two holomorphic prepotentials. In this case the corrections computed by mirror symmetry are precisely analogous to the situation in $N = 2$. For $\tilde{\mathcal{M}}^Q$ the situation is more involved since the geometry of the moduli space changes drastically. Nevertheless we were able to show that mirror symmetry in the large volume - large complex structure limit. However, understanding the nature of the corrections computed by mirror symmetry appear to be more involved and certainly deserves further study.

Acknowledgments

We have greatly benefited from conversations with I. Benmachiche, I. Brunner, V. Cortés, R. Grimm, S. Gukov, H. Jockers, A. Klemm and S. Schäfer-Nameki.

This work is supported by the DFG – The German Science Foundation, the European RTN Program MRTN-CT-2004-503369 and the DAAD – the German Academic Exchange Service.

Appendix

A $N = 2$ special geometry of the Calabi-Yau moduli space

In this appendix we briefly summarize the $N = 2$ special geometry of the Calabi-Yau moduli space. A more detailed discussion can be found, for example, in refs. [16, 61, 62, 37, 63]. A special Kähler manifold \mathcal{M} is a Hodge-Kähler manifold (with line bundle \mathcal{L}) of real dimension $2n$ with associated holomorphic flat $Sp(2n+2, \mathbb{R})$ vector bundle \mathcal{H} over \mathcal{M} . Furthermore there exists a holomorphic section $\Omega(z)$ of \mathcal{L} such that

$$K(z, \bar{z}) = -\ln i\langle \Omega(z), \bar{\Omega}(\bar{z}) \rangle, \quad \langle \Omega, \partial_{z^K} \Omega \rangle = 0, \quad K = 1, \dots, n, \quad (\text{A.1})$$

where K is the Kähler potential of \mathcal{M} and $\langle \cdot, \cdot \rangle$ is the symplectic product on the fibers. This is precisely what one encounters in the moduli space of the complex structure

deformations of a Calabi-Yau manifold with Ω being the holomorphic three-form. In this case one is lead to set $n = h^{(2,1)}$ and identify the fibers of the associated Sp -bundle with $H^3(Y, \mathbb{C})$. The symplectic product is given by the intersections on $H^3(Y, \mathbb{C})$ as

$$\langle \alpha, \beta \rangle = \int_Y \alpha \wedge \beta . \quad (\text{A.2})$$

The Kähler covariant derivatives of Ω are denoted by χ_K as explicitly given in (2.11). In terms of the symplectic basis $(\alpha_{\hat{K}}, \beta^{\hat{K}})$ introduced in (2.6) both Ω and χ_K enjoy the expansion

$$\Omega = Z^{\hat{K}} \alpha_{\hat{K}} - \mathcal{F}_{\hat{K}} \beta^{\hat{K}} , \quad \chi_K = \chi_{\hat{K}}^{\hat{L}} \alpha_{\hat{L}} - \chi_{\hat{L}|K}^{\hat{L}} \beta^{\hat{L}} . \quad (\text{A.3})$$

The holomorphic functions $Z^{\hat{K}}(z)$ and $\mathcal{F}_{\hat{K}}(z)$ are called the periods of Ω , while $\chi_{\hat{K}}^{\hat{L}}(z, \bar{z})$ and $\chi_{\hat{L}|K}^{\hat{L}}(z, \bar{z})$ are the periods of χ_K . In terms of $Z^{\hat{K}}, \mathcal{F}_{\hat{K}}$ the Kähler potential (A.1) can be rewritten as in (2.12).

For every special Kähler manifold there exists a complex matrix $\mathcal{M}_{\hat{K}\hat{L}}(z, \bar{z})$ defined as

$$\mathcal{M}_{\hat{K}\hat{L}} = (\bar{\chi}_{\hat{K}|\bar{M}} \quad \mathcal{F}_{\hat{K}})(\bar{\chi}_{\bar{M}}^{\hat{L}} \quad Z^{\hat{L}})^{-1} , \quad (\text{A.4})$$

where $\chi_{\hat{K}}^{\hat{L}}$ and $\chi_{\hat{L}|K}$ are given in (A.3). Furthermore, one extracts from (A.4) the identities

$$\mathcal{F}_{\hat{K}} = \mathcal{M}_{\hat{K}\hat{L}} Z^{\hat{L}} , \quad \chi_{\hat{L}|K} = \bar{\mathcal{M}}_{\hat{L}\hat{M}} \chi_K^{\hat{M}} , \quad (\text{A.5})$$

which can be used to rewrite (A.1) as

$$\begin{aligned} G_{M\bar{N}} &= -2e^K \chi_M^{\hat{K}} \text{Im} \mathcal{M}_{\hat{K}\hat{L}} \bar{\chi}_{\bar{N}}^{\hat{L}} , \quad 1 = -2e^K Z^{\hat{K}} \text{Im} \mathcal{M}_{\hat{K}\hat{L}} \bar{Z}^{\hat{L}} , \\ 0 &= -2\bar{\chi}_{\bar{M}}^{\hat{K}} \text{Im} \mathcal{M}_{\hat{K}\hat{L}} \bar{Z}^{\hat{L}} . \end{aligned} \quad (\text{A.6})$$

If one assumes that the Jacobian matrix $\partial_{z^L}(Z^K/Z^0)$ is invertible $\mathcal{F}_{\hat{K}}$ is the derivative of a holomorphic prepotential \mathcal{F} with respect to the periods $Z^{\hat{K}}$. It is homogeneous of degree two and obeys

$$\mathcal{F} = \frac{1}{2} Z^{\hat{K}} \mathcal{F}_{\hat{K}} , \quad \mathcal{F}_{\hat{K}} = \partial_{Z^{\hat{K}}} \mathcal{F} , \quad \mathcal{F}_{\hat{K}\hat{L}} = \partial_{Z^{\hat{K}}} \mathcal{F}_{\hat{L}} , \quad \mathcal{F}_{\hat{L}} = Z^{\hat{K}} \mathcal{F}_{\hat{K}\hat{L}} , \quad (\text{A.7})$$

which implies that $\mathcal{F}_{\hat{K}\hat{L}}(Z)$ is invariant under rescalings of $Z^{\hat{K}}$. Notice that \mathcal{F} is only invariant under a restricted class of symplectic transformations and thus depends on the choice of symplectic basis.

The complex matrix $\mathcal{M}_{\hat{K}\hat{L}}$ defined in (A.4) can be rewritten in terms of the periods $Z^{\hat{K}}$ and the matrix $\mathcal{F}_{\hat{K}\hat{L}}(Z)$ as

$$\mathcal{M}_{\hat{K}\hat{L}} = \bar{\mathcal{F}}_{\hat{K}\hat{L}} + 2i \frac{(\text{Im} \mathcal{F})_{\hat{K}\hat{M}} Z^{\hat{M}} (\text{Im} \mathcal{F})_{\hat{L}\hat{N}} Z^{\hat{N}}}{Z^{\hat{N}} (\text{Im} \mathcal{F})_{\hat{N}\hat{M}} Z^{\hat{M}}} . \quad (\text{A.8})$$

Whenever the Jacobian matrix $\partial_{z^L}(Z^K/Z^0)$ is invertible the $Z^{\hat{K}}$ can be viewed as projective coordinates of $\mathbb{P}_{h^{(2,1)}+1}$. Going to a special gauge, i.e. fixing the Kähler transformations (2.15), one introduces special coordinates z^K by setting $z^K = Z^K/Z^0$. Due

to the homogeneity of \mathcal{F} it is possible to define a holomorphic prepotential $f(z)$ which only depends on the special coordinates as

$$\mathcal{F}(Z) = (Z^0)^2 f(z) . \quad (\text{A.9})$$

In terms of f the Kähler potential given in (A.1) reads

$$K = -\ln i|Z^0|^2 [2(f - \bar{f}) - (\partial_K f + \partial_{\bar{K}} \bar{f})(z^K - \bar{z}^K)] . \quad (\text{A.10})$$

The complexified Kähler deformations t^A introduced in (2.7) are special coordinates of a special Kähler manifold. The Kähler potential of the metric G_{AB} given in (2.17) is of the form (A.10) with

$$f(t) = -\frac{1}{6}\mathcal{K}_{ABC}t^A t^B t^C . \quad (\text{A.11})$$

Furthermore, inserting (A.11) into (A.8) using (A.9) one determines the gauge-couplings $\mathcal{N}_{\hat{A}\hat{B}}(t, \bar{t})$ to be

$$\begin{aligned} \text{Re}\mathcal{N} &= \begin{pmatrix} -\frac{1}{3}\mathcal{K}_{ABC}b^A b^B b^C & \frac{1}{2}\mathcal{K}_{ABC}b^B b^C \\ \frac{1}{2}\mathcal{K}_{ABC}b^B b^C & -\mathcal{K}_{ABC}b^C \end{pmatrix} , \\ \text{Im}\mathcal{N} &= -\frac{\mathcal{K}}{6} \begin{pmatrix} 1 + 4G_{AB}b^A b^B & -4G_{AB}b^B \\ -4G_{AB}b^B & 4G_{AB} \end{pmatrix} , \\ (\text{Im}\mathcal{N})^{-1} &= -\frac{6}{\mathcal{K}} \begin{pmatrix} 1 & b^A \\ b^A & \frac{1}{4}G^{AB} + b^A b^B \end{pmatrix} , \end{aligned} \quad (\text{A.12})$$

where G_{AB} is given in (2.17).

B Supergravity with several linear multiplets

In this appendix we briefly discuss the dualization of several massless linear multiplets to chiral multiplets. We only discuss the bosonic component fields and do not include possible couplings to vector multiplets. Our aim is to extract the Kähler potential for the $N = 1, d = 4$ supergravity theory with all linear multiplets replaced by chiral ones. Let us begin by recalling the effective action for a set of linear multiplets (L^λ, D_2^λ) couple to chiral multiplets N^k . It takes the form³⁰

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}R * \mathbf{1} - \tilde{K}_{N^k \bar{N}^l} dN^k \wedge *d\bar{N}^l + \frac{1}{4}\tilde{K}_{L^\kappa L^\lambda} dL^\kappa \wedge *dL^\lambda \\ & + \frac{1}{4}\tilde{K}_{L^\kappa L^\lambda} dD_2^\kappa \wedge *dD_2^\lambda - \frac{i}{2}dD_2^\lambda \wedge (\tilde{K}_{L^\lambda N^k} dN^k - \tilde{K}_{l^\lambda \bar{N}^k} d\bar{N}^k) , \end{aligned} \quad (\text{B.1})$$

where $\tilde{K}(L, N, \bar{N})$ is a function of the scalars L^λ and the chiral multiplets N^k . The kinetic potential \tilde{K} is the analog of the Kähler potential in the sense that it encodes the dynamics of the linear and chiral multiplets. In order to dualize the linear multiplets (L^λ, D_2^λ) into chiral multiplets (L^λ, ξ_λ) one replaces dD_2^λ by the form D_3^λ and adds the term

$$\mathcal{L} \rightarrow \mathcal{L} + \delta\mathcal{L} , \quad \delta\mathcal{L} = -2\tilde{\xi}_\lambda dD_3^\lambda = -2D_3^\lambda \wedge d\tilde{\xi}_\lambda , \quad (\text{B.2})$$

³⁰This action can be obtained by a straight forward generalization of the action for one linear multiplet given in [64].

where $\tilde{\xi}_\lambda(x)$ is a Lagrange multiplier. Eliminating $\tilde{\xi}_\lambda$ one finds that $dD_3^\lambda = 0$ such that locally $D_3^\lambda = dD_2^\lambda$ as required. Alternatively one can consistently eliminate D_3^λ by inserting its equations of motion

$$*D_3^\kappa = 4\tilde{K}^{L^\kappa L^\lambda} \left(d\tilde{\xi}_\lambda + \frac{i}{4} (\tilde{K}_{L^\lambda N^k} dN^k - \tilde{K}_{L^\lambda \bar{N}^k} d\bar{N}^k) \right) \quad (\text{B.3})$$

back into the Lagrangian (B.1). The resulting dual Lagrangian takes the form

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}R * \mathbf{1} - \tilde{K}_{N^k \bar{N}^l} dN^k \wedge *d\bar{N}^l + \frac{1}{4}\tilde{K}_{L^\kappa L^\lambda} dL^\kappa \wedge *dL^\lambda \\ & + 4\tilde{K}^{L^\kappa L^\lambda} \left(d\tilde{\xi}_\kappa - \frac{1}{2}\text{Im}(\tilde{K}_{L^\kappa N^l} dN^l) \right) \wedge * \left(d\tilde{\xi}_\lambda - \frac{1}{2}\text{Im}(\tilde{K}_{L^\lambda N^k} dN^k) \right) . \end{aligned} \quad (\text{B.4})$$

Since we intend to use these results in the effective action for Calabi-Yau orientifolds, we make a further simplification. We demand that the kinetic potential \tilde{K} is only a function of L^λ and the imaginary part of N^k , which we denote by $l^k = \text{Im}N^k$. This implies that all chiral fields N^k admit a Peccei-Quinn shift symmetry acting on the real parts of N^k as it is indeed the case for the orientifold setups. Thus the effective Lagrangian (B.4) simplifies to

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}R * \mathbf{1} - \frac{1}{4}\tilde{K}_{l^k l^l} dN^k \wedge *d\bar{N}^l + \frac{1}{4}\tilde{K}_{L^\kappa L^\lambda} dL^\kappa \wedge *dL^\lambda \\ & + 4\tilde{K}^{L^\kappa L^\lambda} \left(d\tilde{\xi}_\kappa + \frac{1}{4}\tilde{K}_{L^\kappa l^l} d\text{Re}N^l \right) \wedge * \left(d\tilde{\xi}_\lambda + \frac{1}{4}\tilde{K}_{L^\lambda l^k} d\text{Re}N^k \right) . \end{aligned} \quad (\text{B.5})$$

This $N = 1$ Lagrangian is written completely in terms of chiral multiplets and therefore can be derived from a Kähler potential when choosing appropriate complex coordinates N^k and $T_\lambda = (L^\lambda, \tilde{\xi}_\lambda)$. As we will see in a moment, a direct calculation yields that this Kähler potential is the Legendre transform of \tilde{K} with respect to the scalars L^κ . It takes the form

$$K(T, N) = \tilde{K}(L, N - \bar{N}) - 2(T_\kappa + \bar{T}_\kappa)L^\kappa \quad (\text{B.6})$$

where $L^\kappa(N, T)$ is a function of the complex fields N^k, T_λ . This dependence is implicitly given via the definition of the coordinates T_λ

$$T_\lambda = i\tilde{\xi}_\lambda + \frac{1}{4}\tilde{K}_{L^\lambda} . \quad (\text{B.7})$$

However, in order to calculate the Kähler metric, one only needs to determine the derivatives of $L^\kappa(N, T)$ with respect to N^k, T_λ . They are obtained by differentiating (B.7) and simply read

$$\partial L^\kappa / \partial T_\lambda = 2\tilde{K}^{L^\kappa L^\lambda} , \quad \partial L^\kappa / \partial N^l = -\frac{1}{2i}\tilde{K}^{L^\kappa L^\lambda} \tilde{K}_{L^\lambda l^l} . \quad (\text{B.8})$$

Using these identities one easily calculates the first derivatives of the Kähler potential (B.6) as

$$K_{T_\alpha} = -2L^\alpha , \quad K_{N^A} = \frac{1}{2i}\tilde{K}_{l^A} . \quad (\text{B.9})$$

Applying the equations (B.8) once more when differentiating (B.9) one finds the Kähler metric

$$\begin{aligned} K_{T_\alpha \bar{T}_\beta} &= -4\tilde{K}^{L^\alpha L^\beta} , \quad K_{T_\alpha \bar{N}^A} = i\tilde{K}^{L^\alpha L^\beta} \tilde{K}_{L^\beta l^A} , \\ K_{N^A \bar{N}^B} &= \frac{1}{4}\tilde{K}_{l^A l^B} - \frac{1}{4}\tilde{K}_{l^A L^\alpha} \tilde{K}^{L^\alpha L^\beta} \tilde{K}_{L^\beta l^B} , \end{aligned} \quad (\text{B.10})$$

with inverse

$$\begin{aligned} K^{T_\alpha \bar{T}_\beta} &= -\frac{1}{4} \tilde{K}_{L^\alpha L^\beta} + \frac{1}{4} \tilde{K}_{l^A L^\alpha} \tilde{K}^{l^A l^B} \tilde{K}_{L^\beta l^B} , \\ K^{T_\alpha \bar{N}^B} &= -i \tilde{K}^{l^A l^B} \tilde{K}_{l^A L^\alpha} , \quad K^{N^A \bar{N}^B} = 4 \tilde{K}^{l^A l^B} . \end{aligned} \quad (\text{B.11})$$

Finally, one checks that $K(T, N)$ is indeed the Kähler potential for the chiral part of the Lagrangian (B.5). This is done by inserting in the definition of T_κ given in (B.7) and the Kähler metric (B.10) into

$$\mathcal{L} = -\frac{1}{2} R * \mathbf{1} - K_{M^I \bar{M}^J} dM^I \wedge * d\bar{M}^J , \quad (\text{B.12})$$

where $M^I = (N^k, T_\lambda)$.

C Gerneral reduction of the quaternionic space

In this appendix we present a more detailed analysis of the moduli space $\tilde{\mathcal{M}}^Q$, which is a Kähler submanifold in the quaternionic space \mathcal{M}^Q . Our aim is to show that the Kähler potential (3.42) with coordinates T_κ, N^k introduced in (3.47) indeed encode the correct low-energy dynamics of the theory obtained by Kaluza-Klein reduction. Furthermore we show that K^Q always obeys a no-scale type condition equivalent to (3.43). Most of the calculations will be based on the Legendre transform method applied to the real part of the coordinates T_κ . On the level of superfields one can interpret this as dualization of these chiral multiplets into linear multiplets as discussed in appendix B.

Let us start by performing the reduction of the ten-dimensional theory by using the general basis $(\alpha_{\hat{K}}, \beta^{\hat{K}})$ introduced in (3.46). It was chosen such that it splits on $H^3(Y) = H_+^3 \oplus H_-^3$ as

$$(\alpha_k, \beta^\lambda) \in H_+^3(Y) , \quad (\alpha_\lambda, \beta^k) \in H_-^3(Y) , \quad (\text{C.1})$$

where both eigenspaces are spanned by $h^{2,1} + 1$ basis vectors. As remarked above, we will only concentrate on the moduli space $\tilde{\mathcal{M}}^Q$, such that we can set $t^a = 0$ and $A^\alpha = 0$. Due to (3.6), the ten-dimensional three-form \hat{C}_3 is expanded in elements of $H_+^3(Y)$ as

$$C_3 = \xi^k(x) \alpha_k - \tilde{\xi}_\lambda(x) \beta^\lambda , \quad (\text{C.2})$$

where $\xi^k, \tilde{\xi}_\lambda$ are $h^{2,1} + 1$ real space-time scalars in four-dimensions. Inserting this Ansatz into the ten-dimensional effective action one finds

$$\begin{aligned} S_{\tilde{\mathcal{M}}^Q}^{(4)} &= \int -dD \wedge *dD - G_{KL}(q) dq^K \wedge *dq^L + \frac{1}{2} e^{2D} \text{Im} \mathcal{M}_{kl} d\xi^k \wedge *d\xi^l \quad (\text{C.3}) \\ &\quad + \frac{1}{2} e^{2D} (\text{Im} \mathcal{M})^{-1}{}^{\kappa\lambda} (d\tilde{\xi}_\kappa - \text{Re} \mathcal{M}_{\kappa l} d\xi^l) \wedge * (d\tilde{\xi}_\lambda - \text{Re} \mathcal{M}_{\lambda k} d\xi^k) , \end{aligned}$$

where compared to (3.19) only the terms involving $\xi^k, \tilde{\xi}_\lambda$ have changed. The metric $G_{KL}(q)$ was introduced in (3.27) and is the induced metric on the space of real complex structure deformations $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$ parameterized by q^K . It remains to comment on the kinetic and coupling terms of the scalars $\xi^k, \tilde{\xi}_\lambda$. In the quaternionic metric (2.9) of the $N = 2$

theory they couple via the matrix $\mathcal{M}_{\hat{K}\hat{L}}$ given in (2.16). Using the split of the symplectic basis $(\alpha_{\hat{K}}, \beta^{\hat{K}})$ as given in (C.1) and the fact that for $\alpha \in H_+^3, * \alpha \in H_-^3$ one concludes

$$\text{Re}\mathcal{M}_{\kappa\lambda}(q) = \text{Re}\mathcal{M}_{kl}(q) = \text{Im}\mathcal{M}_{\lambda k}(q) = 0 , \quad (\text{C.4})$$

whereas $\text{Re}\mathcal{M}_{k\lambda}, \text{Im}\mathcal{M}_{\kappa\lambda}, \text{Im}\mathcal{M}_{kl}$ are generally non-zero on $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$. The explicit form of non-vanishing components can be obtained by restricting (A.8) to $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$ and using the constraints (3.45).

In order to combine the scalars e^D, q^K with $\xi^k, \tilde{\xi}_\lambda$ into complex variables, we have to redefine these fields and rewrite the first two terms in (C.3). Thus we define the $h^{2,1} + 1$ real coordinates

$$L^\lambda = -e^{2D} \text{Im}[CZ^\lambda(q)] , \quad l^k = \text{Re}[CZ^k(q)] , \quad (\text{C.5})$$

which is consistent with the orientifold constraint (3.45). The additional factor of e^{2D} was included in order to match the dilaton factors later on. Using (C.5) one calculates the Jacobian matrix

$$\mathcal{S} = \begin{pmatrix} \partial L^\lambda / \partial e^{-D} & \partial L^\lambda / \partial q^s & \partial L^\lambda / \partial q^\sigma \\ \partial l^k / \partial e^{-D} & \partial l^k / \partial q^s & \partial l^k / \partial q^\sigma \end{pmatrix} , \quad (\text{C.6})$$

where $q^K = (q^s, q^\sigma)$ are the $h^{(2,1)}$ real coordinates introduced in (3.26). One evaluates the derivatives by applying (3.25) such that

$$\mathcal{S} = \begin{pmatrix} e^{3D} \text{Im}(CZ^\lambda) & -e^{2D} \text{Im}(C\chi_s^\lambda) & -e^{2D} \text{Re}(C\chi_\sigma^\lambda) \\ e^D \text{Re}(CZ^k) & \text{Re}(C\chi_s^k) & -\text{Im}(C\chi_\sigma^k) \end{pmatrix} , \quad (\text{C.7})$$

where $\chi_K^{\hat{L}}$ is defined in (A.3). It is now straight forward to rewrite (C.3) by using the identities (A.6) of special geometry as

$$\begin{aligned} S_{\mathcal{M}^Q}^{(4)} = & \int 2e^{-2D} \text{Im}\mathcal{M}_{\kappa\lambda} dL^\kappa \wedge *dL^\lambda + 2e^{2D} \text{Im}\mathcal{M}_{kl} dl^k \wedge *dl^l + \frac{e^{2D}}{2} \text{Im}\mathcal{M}_{kl} d\xi^k \wedge *d\xi^l \\ & + \frac{e^{2D}}{2} (\text{Im}\mathcal{M})^{-1}{}^{\kappa\lambda} (d\tilde{\xi}_\kappa - \text{Re}\mathcal{M}_{\kappa k} d\xi^k) \wedge * (d\tilde{\xi}_\lambda - \text{Re}\mathcal{M}_{\lambda k} d\xi^k) . \end{aligned} \quad (\text{C.8})$$

From (C.8) one sees that the scalars l^k and ξ^k nicely combine into complex coordinates

$$N^k = \frac{1}{2}\xi^k + il^k = \frac{1}{2}\xi^k + i\text{Re}(CZ^k) , \quad (\text{C.9})$$

which corresponds to (3.47). In contrast, one observes that the metric for the kinetic terms of the scalars $\tilde{\xi}_\lambda$ is exactly the inverse of the one appearing in the kinetic terms of the scalar fields L^λ . This hints to the fact that the Lagrangian (C.8) is obtained by dualizing a set of linear multiplets (L^λ, D_2^λ) into chiral multiplets $(L^\lambda, \tilde{\xi}_\lambda)$. The effective action of several linear multiplets coupled to a set of chiral multiplets N^k is given in equation (B.1). In analogy to the Kähler potential in the standard $N = 1$ supergravity the couplings and kinetic terms of the linear and chiral multiplets are encoded by a single real function $\tilde{K}(L, N, \bar{N})$ [64]. Dualizing the massless two-forms D_2^λ to scalar fields $\tilde{\xi}_\lambda$ as described in appendix B the resulting effective action in terms of $(L^\lambda, \tilde{\xi}_\lambda)$ and N^k takes the form (B.5). This implies that (C.8) is indeed obtained from (B.5), when appropriately

specifying the function \tilde{K} . To extract $\tilde{K}(L, N, \bar{N})$ we compare the action (C.8) with (B.5) and read off the metric

$$\tilde{K}_{L^\kappa L^\lambda} = 8e^{-2D} \text{Im} \mathcal{M}_{\kappa\lambda}, \quad \tilde{K}_{l^k l^l} = -8e^{2D} \text{Im} \mathcal{M}_{kl}, \quad \tilde{K}_{L^\kappa l^l} = -8 \text{Re} \mathcal{M}_{\kappa l}, \quad (\text{C.10})$$

where we have used that the metric is independent of $\xi^k, \tilde{\xi}_\lambda$. This metric can be obtained from a kinetic potential of the form

$$\tilde{K}(L, l) = -\ln [e^{-4D}] + 8e^{2D} \text{Im} [\rho^* \mathcal{F}(CZ^k)], \quad (\text{C.11})$$

where \mathcal{F} is the prepotential of the special Kähler manifold \mathcal{M}^{cs} restricted to the real subspace $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$. The map ρ was given in (3.26) and enforces the constraints (3.45). To show that \tilde{K} indeed yields the correct metric (C.10) one differentiates (C.11) with respect to e^{-D}, q^K and uses the inverse of \mathcal{S} . Applying equations (A.5) one finds its first derivatives

$$\tilde{K}_{L^\lambda} = -8 \text{Re} [C\mathcal{F}_\lambda(q)] \quad \tilde{K}_{l^k} = 8e^{2D} \text{Im} [C\mathcal{F}_k(q)]. \quad (\text{C.12})$$

Repeating the procedure and differentiating (C.12) with respect to e^{-D}, q^K and using \mathcal{S}^{-1} one can apply (A.4) to show (C.10).

As explained in appendix B the actual Kähler potential of $\tilde{\mathcal{M}}^Q$ is the Legendre transform of \tilde{K} with respect to the variables L^λ . There we also found the explicit definition of the complex coordinates T_λ combining $(L^\lambda, \tilde{\xi}_\lambda)$. Thus the Kähler potential $K^Q(T, N)$ is obtained from $\tilde{K}(L, N)$ by setting

$$K^Q(T, N) = \tilde{K}(L, N) - 2(T_\lambda + \bar{T}_\lambda)L^\lambda, \quad (\text{C.13})$$

where $L^\lambda(T + \bar{T}, N, \bar{N})$ is now a function of the chiral multiplets T_λ and N^k . This dependence is implicitly defined via the equation

$$T_\lambda + \bar{T}_\lambda = \frac{1}{2}\tilde{K}_{L^\lambda}. \quad (\text{C.14})$$

Using (C.12) and fixing the normalization of the imaginary part of T_λ by comparing (C.8) with (B.5) one finds

$$T_\lambda = i\tilde{\xi}_\lambda + \frac{1}{4}\tilde{K}_{L^\lambda} = i\tilde{\xi}_\lambda - 2 \text{Re}(CF_\lambda), \quad (\text{C.15})$$

which coincides with (3.47) already quoted in section 3.3. To give an explicit expression for K^Q we plug equation (C.11) into (C.13). Inserting the $N = 2$ identity $\mathcal{F} = \frac{1}{2}Z^{\hat{K}}\mathcal{F}_{\hat{K}}$, the constraint equations (3.45) and (C.5), (C.12) we rewrite

$$K^Q = -\ln e^{-4D} + \frac{1}{2}(l^k \tilde{K}_{l^k} - L^\lambda \tilde{K}_{L^\lambda}). \quad (\text{C.16})$$

It is possible to evaluate the terms appearing in the parentheses. In order to do that we combine the equations (C.5) and (C.12) to the simple form

$$\text{Re}(C\Omega) = l^k \alpha_k + \frac{1}{8}\tilde{K}_{L^\lambda} \beta^\lambda, \quad e^{2D} \text{Im}(C\Omega) = -L^\lambda \alpha_\lambda - \frac{1}{8}\tilde{K}_{l^k} \beta^k. \quad (\text{C.17})$$

We now use equation (2.12) and the definition (3.36) of C to calculate

$$2 \int_Y \text{Re}(C\Omega) \wedge \text{Im}(C\Omega) = i \int_Y C\Omega \wedge \overline{C\Omega} = e^{-2D}. \quad (\text{C.18})$$

Inserting the equations (C.17) into (C.18) we find

$$L^\lambda \tilde{K}_{L^\lambda} - l^k \tilde{K}_{l^k} = 4 . \quad (\text{C.19})$$

Inserting this constraint into (C.16) we have shown that the Kähler potential has indeed the form (3.42).³¹ Moreover, (C.19) directly translates into a no-scale type condition for K^Q

$$K_{w^{\hat{K}}} K^{w^{\hat{K}} \bar{w}^{\hat{L}}} K_{\bar{w}^{\hat{L}}} = 4 , \quad (\text{C.20})$$

where $w^{\hat{K}} = (T_\kappa, N^k)$. In order to see this, one inserts the inverse Kähler metric (B.11), the Kähler derivatives (B.9) and the derivatives of (C.19) back into (C.19). In other words, we were able to translate one of the special Kähler conditions present in the underlying $N = 2$ theory into a constraint on the geometry of $\tilde{\mathcal{M}}^Q$.

Let us end our discussion of $\tilde{\mathcal{M}}^Q$, by giving two specific examples for \tilde{K} satisfying the constraint (C.19), namely

$$\begin{aligned} \tilde{K}_1 &= \ln \left[a_1 \frac{\kappa_{\kappa\lambda\mu} L^\kappa L^\lambda L^\mu}{l^0} \right] + b_1 \frac{\kappa_{\kappa k l} L^\kappa l^k l^l}{l^0} , \\ \tilde{K}_2 &= -\ln \left[a_2 \frac{\kappa_{k l m} l^k l^l l^m}{L^0} \right] + b_2 \frac{\kappa_{\kappa\lambda l} L^\kappa L^\lambda l^l}{L^0} , \end{aligned} \quad (\text{C.21})$$

where $a_{1,2}, b_{1,2}$ are some constants. In [21] it was shown, that \tilde{K}_1 is the correct potential describing the dynamics of IIB orientifolds with $O3/O7$ planes. On the other hand, \tilde{K}_2 is the correct potential for IIB orientifolds with $O5/O9$ planes. $\tilde{K}_{1,2}$ have this simple form since instanton corrections are not taken into account.

D The Geometry of the moduli space of CY orientifolds

In this section we give an alternative formulation of the geometric structures of the moduli space $\tilde{\mathcal{M}}^Q$ which is closely related the moduli space of supersymmetric Lagrangian submanifolds in a Calabi-Yau threefold [23].³² In this set-up also the no-scale conditions (3.43), (C.19) are interpreted geometrically.

In section 3.3 we started from an $N = 2$ quaternionic manifold \mathcal{M}^Q and determined the submanifold $\tilde{\mathcal{M}}^Q$ by imposing the orientifold projection. $N = 1$ supersymmetry ensured that this submanifold is Kähler. \mathcal{M}^Q has a second but different Kähler submanifold \mathcal{M}^{cs} which intersects with $\tilde{\mathcal{M}}^Q$ on the real manifold $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$. The c-map is in some sense the reverse operation where \mathcal{M}^Q is constructed starting from \mathcal{M}^{cs} and shown to be quaternionic [40, 17]. In this appendix we analogously construct the Kähler manifold $\tilde{\mathcal{M}}^Q$ starting from $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$.

³¹By using the equation (C.18) and $*\Omega = -i\Omega$ it is straight forward to show $e^{-2D} = 2 \int \text{Re}(C\Omega) \wedge *\text{Re}(C\Omega)$

³²This analysis can equivalently be applied to the moduli space of G_2 compactifications of M-theory.

In fact the proper starting point is not $\mathcal{M}_{\mathbb{R}}^{\text{cs}}$ but rather $\mathcal{M}_{\mathbb{R}} = \mathcal{M}_{\mathbb{R}}^{\text{cs}} \times \mathbb{R}$ which is the local product of the moduli space of real complex structure deformations of a Calabi-Yau orientifold times the real dilaton direction.³³ Its local geometry is encoded in the variations of the real and imaginary part of the normalized holomorphic three-form $C\Omega$. This form naturally defines an embedding

$$E : \mathcal{M}_{\mathbb{R}} \rightarrow V \times V^* = H_+^3(\mathbb{R}) \times H_-^3(\mathbb{R}) . \quad (\text{D.1})$$

where $V = H_+^3(\mathbb{R})$ and we used the intersection form $\langle \alpha, \beta \rangle = \int \alpha \wedge \beta$ on $H^3(Y)$ to identify $V^* \cong H_-^3(\mathbb{R})$. $V \times V^*$ naturally admits a symplectic form \mathcal{W} and an indefinite metric \mathcal{G} defined as

$$\begin{aligned} \mathcal{W}((\alpha_+, \alpha_-), (\beta_+, \beta_-)) &= \langle \alpha_+, \beta_- \rangle - \langle \beta_+, \alpha_- \rangle , \\ \mathcal{G}((\alpha_+, \alpha_-), (\beta_+, \beta_-)) &= \langle \alpha_+, \beta_- \rangle + \langle \beta_+, \alpha_- \rangle , \end{aligned} \quad (\text{D.2})$$

where $\alpha_{\pm}, \beta_{\pm} \in H_{\pm}^3(\mathbb{R})$.

Now we construct E in such a way that $\mathcal{M}_{\mathbb{R}}$ is a Lagrangian submanifold of $V \times V^*$ with respect to \mathcal{W} and its metric is induced from \mathcal{G} , i.e.

$$E^*(\mathcal{W}) = 0 , \quad E^*(\mathcal{G}) = g \quad (\text{D.3})$$

where

$$\frac{1}{2}g = dD \otimes dD + G_{KL}dq^K \otimes dq^L . \quad (\text{D.4})$$

is the metric on $\mathcal{M}_{\mathbb{R}}$ as determined in (3.19). As we are going to show momentarily E is given by

$$E(q^{\hat{K}}) = (2\text{Re}(C\Omega), -2e^{2D}\text{Im}(C\Omega)) , \quad (\text{D.5})$$

where $q^{\hat{K}} = (e^{-D}, q^K)$ and Ω is evaluated at $q^K \in \mathcal{M}_{\mathbb{R}}^{\text{cs}}$. Additionally E satisfies

$$\mathcal{G}(E(q^{\hat{K}}), E(q^{\hat{K}})) = 4 , \quad (\text{D.6})$$

for all q^K . This implies that the image of all points in $\mathcal{M}_{\mathbb{R}}$ have the same distance from the origin. Later on we will show that this translates into the no-scale condition (C.20).

Before we do so let us first show that the E given in (D.5) indeed satisfies (D.3) and (D.6). The explicit calculation is straight forward and essentially included in the reduction presented in appendix C.³⁴ To see this we express the map E defined in (D.5) and the conditions in terms of the basis $(\alpha_{\hat{K}}, \beta^{\hat{K}})$ introduced in (C.1). We use eq. (C.17) and expand

$$E(q^{\hat{K}}) = (2l^k\alpha_k + \frac{1}{4}\tilde{K}_{L^{\kappa}}\beta^{\kappa}, 2L^{\kappa}\alpha_{\kappa} + \frac{1}{4}\tilde{K}_{l^k}\beta^k) , \quad (\text{D.7})$$

³³The $N = 2$ analog of $\mathcal{M}_{\mathbb{R}}$ is the extended moduli space $\hat{\mathcal{M}}^{\text{cs}} = \mathcal{M}_{\mathbb{R}}^{\text{cs}} \times \mathbb{C}$ where \mathbb{C} is the complex line normalizing Ω . The corresponding modulus can be identified with the complex dilaton [47]. The orientifold projection fixes the phase of the complex dilaton (it projects out the four-dimensional B_2) to be θ and thus reduces \mathbb{C} to \mathbb{R} .

³⁴Formally one has to first evaluate $E_*(\partial_{Q^{\hat{K}}})$ and expresses the result in terms of the $(3,0)$ -form Ω and the $(2,1)$ -forms χ_K . One then uses that by definition of the pullback $E^*\omega(\partial_{q^{\hat{K}}}, \cdot) = \omega(E_*(\partial_{q^{\hat{K}}}), \cdot)$ for a form ω on $V \times V^*$. Applied to \mathcal{G} and \mathcal{W} one finds that the truncation of the special Kähler (2.12) and (2.10) indeed imply (D.3). This calculation does not make use of any specific basis of H_{\pm}^3 .

where l^k, L^κ and $\tilde{K}_{L^\kappa}, \tilde{K}_{l^k}$ are functions of $q^{\hat{K}}$ as given in (C.5) and (C.12). We define coordinates $u^{\hat{K}} = (2l^k, \frac{1}{4}\tilde{K}_{L^\kappa})$ on V and coordinates $v_{\hat{K}} = (\frac{1}{4}\tilde{K}_{l^k}, -2L^\kappa)$ on V^* . In these coordinates the first two conditions in (D.3) simply read

$$E^*(du^{\hat{K}} \wedge dv_{\hat{K}}) = 0, \quad E^*(du^{\hat{K}} \otimes dv_{\hat{K}}) = g. \quad (\text{D.8})$$

From appendix C we further know that $\tilde{K}_{L^\kappa}, \tilde{K}_{l^k}$ are derivatives of a kinetic potential \tilde{K} and thus we can evaluate $du^{\hat{K}}$ and $dv_{\hat{K}}$ in terms of l^k, L^κ . Inserting the result into (D.8) the second equation can be rewritten as

$$\frac{1}{2}g = \frac{1}{4}\tilde{K}_{l^k l^l} dl^k \otimes dl^l - \frac{1}{4}\tilde{K}_{L^\kappa L^\lambda} dL^\kappa \otimes dL^\lambda, \quad (\text{D.9})$$

while the first equation is trivially fulfilled due to the symmetry of $\tilde{K}_{l^k l^l}$ and $\tilde{K}_{L^\kappa L^\lambda}$. This metric is exactly the one appearing in the action (C.8) when using (C.10). Expressing g in coordinates e^D, q^K leads to (D.4), as we have already checked by going from (C.3) to (C.8) above. Furthermore, inserting (D.7) into (D.6) it exactly translates into the no-scale condition (C.19), which was shown in appendix C to be equivalent to (3.43).

We have just shown that $\mathcal{M}_{\mathbb{R}}$ is a Lagrangian submanifold of $V \times V^*$. Identifying $T^*V \cong V \times V^*$ we conclude that $\mathcal{M}_{\mathbb{R}}$ can be obtained as the graph $(\alpha(u), u)$ of a closed one-form α . This implies that we can locally find a generating function $K' : V \rightarrow \mathbb{R}$ such that $\alpha = dK'$. In local coordinates $(v_{\hat{K}}, u^{\hat{K}})$ this amounts to

$$v_{\hat{K}} = \frac{\partial K'}{\partial u^{\hat{K}}} \quad (\text{D.10})$$

such that

$$-L^\kappa(u) = 2 \frac{\partial K'(u)}{\partial \tilde{K}_{L^\kappa}}, \quad \tilde{K}_{l^k}(u) = 2 \frac{\partial K'(u)}{\partial l^k}. \quad (\text{D.11})$$

These equations are satisfied if we define K' in terms of \tilde{K} as

$$2K' = \tilde{K}(L(u), l) - \tilde{K}_{L^\kappa}(u) L^\kappa(u), \quad (\text{D.12})$$

which is nothing but the Legendre transform of \tilde{K} with respect to L^κ . Later on we show that the function $2K'$ is identified with the Kähler potential K given in (3.42).

In order to do that, we now extend our discussion to the full moduli space $\tilde{\mathcal{M}}^Q$ including the scalars $\zeta^{\hat{K}} = (\xi^k, \tilde{\xi}_\kappa)$ parameterizing the three-form \hat{C}_3 in $H_+^3(\mathbb{R})$. Locally one has

$$\tilde{\mathcal{M}}^Q = \mathcal{M}_{\mathbb{R}} \times H_+^3(\mathbb{R}). \quad (\text{D.13})$$

The tangent space at a point p in $\tilde{\mathcal{M}}^Q$ can be identified as

$$T_p \tilde{\mathcal{M}}^Q \cong H_+^3(\mathbb{R}) \oplus H_+^3(\mathbb{R}) \cong H_+^3(\mathbb{R}) \otimes \mathbb{C}, \quad (\text{D.14})$$

where the first isomorphism is induced by the embedding E given in (D.5). This is a complex vector space and thus $\tilde{\mathcal{M}}^Q$ admits an almost complex structure I . In components it is given by

$$I(\partial_{q^{\hat{K}}}) = (\partial u^{\hat{L}} / \partial q^{\hat{K}}) \partial_{\zeta^{\hat{L}}}, \quad I((\partial u^{\hat{L}} / \partial q^{\hat{K}}) \partial_{\zeta^{\hat{L}}}) = -\partial_{q^{\hat{K}}}, \quad (\text{D.15})$$

where we have used that I is induced by the embedding map E . One can show that the almost complex structure I is integrable, since

$$dw^{\hat{K}} = du^{\hat{K}} + id\zeta^{\hat{K}} = (\partial u^{\hat{L}}/\partial q^{\hat{K}})dq^{\hat{K}} + id\zeta^{\hat{K}} , \quad (\text{D.16})$$

are a basis of $(1,0)$ forms and $w^{\hat{K}} = u^{\hat{K}} + i\zeta^{\hat{K}}$ are complex coordinates on $\tilde{\mathcal{M}}^Q$. Using the definition of $u^{\hat{K}}$ one infers that as expected $w^{\hat{K}} = (N^k, T_\kappa)$. Moreover, one naturally extends the metric g on $T\mathcal{M}_\mathbb{R}$ to a hermitian metric on $T\tilde{\mathcal{M}}^Q$. The corresponding two-form is then given by

$$\tilde{\omega}(\partial_{\zeta^{\hat{L}}}, \partial_{q^{\hat{K}}}) = g(I\partial_{\zeta^{\hat{L}}}, \partial_{q^{\hat{K}}}) , \quad \tilde{\omega}(\partial_{\zeta^{\hat{K}}}, \partial_{\zeta^{\hat{L}}}) = \tilde{\omega}(\partial_{q^{\hat{K}}}, \partial_{q^{\hat{L}}}) = 0 . \quad (\text{D.17})$$

Using the definition (D.15) of the almost complex structure and equation (D.3), one concludes that $\tilde{\omega}$ is given by

$$\tilde{\omega} = dv_{\hat{K}} \wedge d\zeta^{\hat{K}} = 2i \frac{\partial^2 K'}{\partial w^{\hat{K}} \partial \bar{w}^{\hat{L}}} dw^{\hat{K}} \wedge d\bar{w}^{\hat{L}} , \quad (\text{D.18})$$

where for the second equality we applied (D.10) and expressed the result in coordinates $w^{\hat{K}} = u^{\hat{K}} + i\zeta^{\hat{K}}$. Note that K' is a function of $u^{\hat{K}}$ only, such that derivatives with respect to $w^{\hat{K}}$ translate to the ones with respect to $u^{\hat{K}}$. Equation (D.18) implies that $K^Q = 2K'$ is indeed the correct Kähler potential for the moduli space $\tilde{\mathcal{M}}^Q$.

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